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TECHNICAL NOTE

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FURTHER DEVELOPMENTS ON THE REQUIRED NUMBER OF RANDOMLY SPACED COMMUNICATION AND NAVIGATION SATELLITES

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SUMMARY

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This study is a continuation of the research problem presented in NASA Technical Note D-619 and is limited to near-earth satellites deployed singly in randomly spaced circular orbits. Certain geometric aspects of the communication and navigation satellite concepts are investigated.

Results of two systematic studies are presented from which estimates of the required number of satellites for many practical communication and navigation links can be obtained. The primary difference between the two studies is the shape of the region of mutual communication; both lenticular and circular shapes were studied.

Results for a sample worldwide communications system, in which the lenticular region of mutual communication is used, are also presented. The communication links for this system are chosen primarily for intercontinental and large city communications. A total of 70 links was studied. For combinations of minimum station elevation angle for communicating with the satellites and orbit altitude where the angular diameter of the circular region of communication at each station is 118°, it was found that 45 satellites in orbits inclined 80° to the equator would allow communication (a) over 99.9 percent of the time over 51 percent of the links, (b) 99.9 to 99.0 percent of the time over an additional 33 percent of the links, and (c) 99.0 to 97.0 percent of the time over the remaining 16 percent of the links.

For a worldwide navigation satellite system (or communication system based on circular regions of mutual communication) it was found that, for the range of circle diameters considered, the optimum orbit inclination angle lies between 53° and 64°.

INTRODUCTION

With the success of the NASA ECHO I (1960 Iota 1) and the U.S. Army Courier 1B (1960 Nu 1) satellites, the feasibility of the satellite concept for worldwide communications has been aptly demonstrated. There are basically three different types of satellite communication systems: the low-altitude passive satellites (such as ECHO I), the low-altitude active satellites (such as Courier), and the 22,300-mile-altitude "stationary" active satellites (24-hour equatorial orbit). Various aspects of these systems have been studied by several authors. For example, in reference 1 problems encountered in the ECHO I passive system are discussed. In references 2 and 3 the electronic network requirements and the geometric aspects for the low-altitude systems are discussed on a limited basis. Also, in reference 3 as well as in reference 4 some aspects of interplanetary communications utilizing satellites are presented. In reference 5 the 24-hour-orbit active system for global communications is discussed. In references 6 and 7 the geometric aspects only for the low-altitude systems are studied. In the latter two reports methods based on a lenticular region of mutual communication (ref. 6) and on a circular region (ref. 7) were developed to determine the minimum number of randomly spaced satellites required for nearly interruption-free communication time between two ground stations. Results of further study of these geometric aspects are presented herein.

L1858

Reference 6 was primarily concerned with the determination of the geometric parameters involved in the communication satellite problem and gave results for a sample communication link only. The primary purpose of the present report is to give the results of a systematic study, utilizing the method of reference 6, from which estimates of the required number of satellites for many practical communication links can be obtained. The satellites are considered to be distributed singly in randomly spaced circular orbits. Results are also presented for a sample worldwide communications system.

In addition, the results of a systematic study to determine the required number of communication satellites for circular regions of mutual communication, a special case of the preceding study, are presented in chart form. This study is presented primarily for communication between the satellite and one ground station, for example, navigation satellites. However, it has been pointed out in reference 7 that communication between two stations might also be analyzed on this basis. In reference 7 the region of mutual communication is assumed to be circular because of the effects of the signal-to-noise ratio on transmission properties. The study discussed herein covers a range of circle diameters which complements the range covered by reference 7. Also, the present study gives off-optimum as well as optimum results in order that compromise results can be obtained for communicating over several links with a common set of satellites.

SYMBOLS

In this paper, distances are measured in U.S. statute miles (1 U.S. statute mile = 1.60935 kilometers.)

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h	satellite orbit altitude, U.S. statute miles
i	orbit inclination angle with respect to equator, deg
N	number of satellites
$N_{0.99}$	number of satellites for communicating 99 percent of time
$P_c(1)$	probability of communicating having only one satellite in orbit
$P_c(N)$	probability of communicating having N-satellites in orbit
R	mean radius of earth, 3,960 U.S. statute miles
Y,Z	coordinate axes referenced to North Pole (see fig. 2)
Y',Z'	coordinate axes referenced to ground station (see fig. 2)
β	minimum station elevation angle for communicating with satellite, deg
θ	latitude, positive north, deg or radians

$\theta_o' = \beta + \sin^{-1}\left(\frac{R \cos \beta}{R + h}\right)$, deg; see figure 2

θ_d'	angular diameter of region of communication for one station, deg
λ	longitude of ground station, deg
$\Delta\lambda$	change in longitude between ground stations, deg
ϕ	longitude, positive west, deg or radians
$\Delta\phi$	change in longitude across region of mutual communication, radians
$\Delta\phi_o$	change in longitude across region of mutual communication at equator, radians
ψ	colatitude of ground station, deg

Subscripts:

min	minimum
max	maximum
opt	optimum

GEOMETRIC AND PROBABILITY CONSIDERATIONS

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The procedure of reference 6 for solving for the required number of communication satellites is reviewed in this section. This problem has three aspects: First, the region of mutual communication - that is, the sector of sky wherein the satellite must be in order to transmit immediately the signals (by passive reflection or active amplification) between the two selected ground stations - is determined; second, the percent of time or probability that a single satellite is in the region is determined; and, third, this single-satellite probability is used to determine the number of satellites required to assure communication with at least one of N-satellites for a given percentage of total time.

Region of Mutual Communication

In figure 1 are shown two communication stations, located at points A and B on the rotating spherical earth, with the capability of sending signals to and receiving signals from an earth satellite along line-of-sight paths anywhere within a region above some minimum station elevation angle $\beta \geqq 0^{\circ}$ (see inset in fig. 1) and through azimuth angles of 0° to 360°. Thus, for each station the communication paths to the satellite are contained within a conical region with an apex angle at the station of $180^{\circ} - 2\beta$. Each cone may be visualized as intersecting the sphere of satellite orbit altitude and thereby a circular boundary is formed wherein communication between the satellite and the station is possible. Thus, the portion of the surface of the satellite orbital sphere common to both circular boundaries (lenticular shape) is then a region of mutual communication - that is, within this region the satellite can communicate with both stations simultaneously. The equation defining the boundary of the regions of communication is given in reference 6 as (see fig. 2)

$$\phi = \lambda \pm \cos^{-1}\left(\frac{\sin \theta_o' - \sin \theta \cos \psi}{\cos \theta \sin \psi}\right) \qquad (1)$$

for

$$\theta_o' - \psi \leqq \theta \leqq \theta_o' + \psi$$

and

$$\psi \neq 0$$

where

$$\theta_o' = \beta + \sin^{-1}\left(\frac{R \cos \beta}{R + h}\right) \tag{2}$$

(For $\psi = 0$, that is, a ground station at the North or South Pole, the region of communication is simply a circle of constant latitude, $\theta = \theta_o'$.) The region of mutual communication can then be obtained by plotting equation (1) for both stations - that is, for λ and ψ of each ground station - and, thus, the lenticular region common to both circles can be determined.

In some instances it may be impractical to utilize the entire lenticular region of mutual communication. For example, in reference 7, because of the effects of signal-to-noise ratio on transmission properties, the region of mutual communication is assumed to be circular. Also, for communication between the satellite and only one ground station - for example, navigation satellite - the region of mutual communication is circular and hence is defined completely by equation (1).

Probability of Communicating Having Only One Satellite in Orbit

Once the region of mutual communication has been found the probability of communicating between two ground stations having only one satellite in orbit can be determined. This probability is given in reference 6 as

$$P_c(1) = \frac{1}{2\pi^2} \int_{\theta_1}^{\theta_2} \frac{\Delta\phi \cos \theta \, d\theta}{\sqrt{\sin^2 i - \sin^2 \theta}} \tag{3}$$

where the limits of integration θ_1 and θ_2 in radians are the lowest and highest latitudes, respectively, in the boundary of the region of mutual communication unless this region extends to latitudes above $+i$ or below $-i$, in which case the limits become $+i$ or $-i$, respectively. When $\theta = i$, the integrand becomes infinite; however, the integral is still finite. (See ref. 6.)

For $i = 0^{\circ}$ equation (3) does not apply because no integration is required since the satellite does not deviate from the equator. The expression for the probability of communicating having only one satellite in orbit is simply

L1858

$$P_c(1) = \frac{\Delta\phi_o}{2\pi} \tag{4}$$

Required Number of Communication Satellites

Once $P_c(1)$ has been determined, the number of satellites required for communicating for a specified percentage of total time can be obtained from the laws of probability. In the present study as in reference 6 the satellites are assumed to be deployed singly in randomly spaced orbits. Thus, from reference 6, the number of satellites N required for communicating for a specified fraction of total time $P_c(N)$ is

$$N = \frac{\log\left[1 - P_c(N)\right]}{\log\left[1 - P_c(1)\right]} \tag{5}$$

for satellites in circular orbits with equal altitudes and inclination angles.

SCOPE OF CALCULATIONS

As mentioned previously, the primary purpose of this report is to present results of two systematic studies, utilizing the method of reference 6, from which the required number of communication and navigation satellites can be determined. (This method has been programed on an IBM 7090 electronic data processing system.) The basic difference between the two studies is the shape of the region of mutual communication, namely, lenticular and circular. The lenticular region is used for communicating between two ground stations. The circular region is used primarily for communicating between the satellite and only one ground station (for example, navigation satellites) but may also be applied, in some instances, between two stations. For example, in reference 7 the region of mutual communication between two ground stations is assumed to be circular because of the effects of signal-to-noise ratio on transmission properties. The range of parameters for each study is presented in this section.

Study Based on Lenticular Region

The parameters of interest in this study are satellite orbit altitude, minimum station elevation angle for communicating with satellite, station separation distance, station latitude, and direction of links. The first two parameters, satellite altitude and minimum station elevation angle for communicating, may be combined into one variable, namely, the angular diameter of the region of communication for one station, $\theta_d' = 180^\circ - 2\theta_o'$. (See fig. 2 and eq. (2).) In the present study, calculations were made for orbit altitudes of 1,000, 2,000, 3,000, and 5,000 miles at each elevation angle of 0°, 5°, and 10° in order to show the variations of these parameters separately, inasmuch as different factors are involved in choosing the range of each. However, by using equation (2) and the formula for θ_d' for a range of h and β, which includes the preceding values, then the 12 cases for which calculations are made can be shown to be valid for other combinations of h and β. A plot showing these combinations is presented in figure 3. The vertical dashed lines represent the combinations of h and β for which the present calculations were made.

The values of the station separation distances and the direction of the links considered herein are listed in the following table which shows the group designation for these variables:

Station separation distance		Group designation for link direction of -				
Degrees	Miles	90° inclination (North-South)	0° inclination (East-West)	30° inclination	45° inclination	60° inclination
30	2,073	A	E	I	M	Q
45	3,109	B	F	J	N	R
60	4,146	C	G	K	O	S
75	5,182	D	H	L	P	T

For the North-South and East-West links the station latitude is varied from 0° to 90° in 15° intervals. (See table I and fig. 4.) The remainder of the links have been set up so that the latitude of one station is always at 15°, 30°, or 45° in order to have a common basis for determining the effect of the direction of the link. (Again, see table I and fig. 4.)

It should be noted that there is no region of mutual communication when the diameter of the region of single station communication θ_d' is less than or equal to the angular station separation distance. Thus, for the separation distance of 60° the cases for $h = 1{,}000$ miles and $\beta = 10^{\circ}$ are omitted and for the separation distance of 75° the cases for $h = 1{,}000$ miles and $\beta = 0^{\circ}$, 5°, and 10° are omitted.

Study Based on Circular Region

The parameters of interest in this study are satellite orbit altitude, minimum station elevation angle for communicating with satellite, and station latitude. As in the preceding study, the first two parameters can be combined into one variable, the angular diameter of the region of communication θ_d', as shown in figure 3. The same combinations of h and β used in the study based on a lenticular region are used in this study. (See vertical dashed lines in fig. 3.) Also, these combinations of h and β correspond to angular diameters which complement the range of diameters studied in reference 7. The station latitude (or latitude of the center of the circle) is varied from 0° to 90° in 15° intervals.

RESULTS AND DISCUSSION

Systematic Study Based on Lenticular Region

The results of this study are presented in figure 5 in the form of plots showing the variation of the probability of communicating having a single satellite in orbit $P_c(1)$ with orbit inclination angle i. (These curves are labeled for particular combinations of h and β, in order to show the variation with each variable rather than the circle diameter; however, these curves apply to other combinations of h and β, as shown in fig. 3.) This type of plot allows for the determination of the maximum value of $P_c(1)$ and the optimum orbit inclination angle for any given communication link. Also, from this type of plot, a solution can be obtained for the more practical problem of communicating over several links with a common set of satellites by comparing these curves so as to determine a compromise value of i and $P_c(1)$ for these links. Once the desired value of $P_c(1)$ is found, the required number of satellites deployed singly in randomly spaced orbits can be obtained from equation (5) or figure 6.

Effect of various parameters.- The parameters of interest in the study based on a lenticular region of mutual communication are satellite orbit altitude, minimum station elevation angle for communicating with satellite, station separation distance, station latitude, and direction of the communication link. The effects of these parameters on $P_c(1)$ and i are illustrated in this section.

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Orbit altitude and minimum station elevation angle: The effect of h and β is as expected from figure 3; that is, θ_d', and consequently $P_c(1)$, increases with h and decreases with β. For example, the results are practically the same for $h = 3{,}000$ miles and $\beta = 0^{\circ}$ as for $h = 5{,}000$ miles and $\beta = 10^{\circ}$. Thus, if communications can be accomplished down to the horizon with a satellite at an altitude of 3,000 miles and only down to 10° above the horizon with a satellite at an altitude of 5,000 miles, then there is no advantage in going to the higher altitude.

Latitude: From a comparison of the curves of figure 5 that have the same value of station separation distance, the effect of station latitude is shown to have a direct bearing on $P_c(1)$. For example, in figure 7 the regions of mutual communication are shown for cases B-2, N-1, and R-5. By studying this figure and the same cases in figure 5 it can be seen that abrupt changes in $P_c(1)$ occur at a value of i in the vicinity of the maximum and minimum latitudes of the boundary of the regions of mutual communication. Also, it can be seen that, whenever the region extends beyond the polar region (case R-5) or includes a substantial portion of the equator (case B-2), then polar or equatorial orbits, respectively, are the best.

Station separation distance: To illustrate the effect of station separation distance, consider the North-South and East-West links sketched in figure 4(e). Sample results showing i_{opt} and N for 99 percent communication time and three values of θ_d' are presented in table II. From these sample results it is evident that the required number of satellites increases rapidly with station separation distance for the lower values of θ_d'. However, for larger values of θ_d' this penalty is reduced considerably. In fact, for the four southern links in figure 4(e) there is no change in i_{opt} or the number of satellites at the higher values of θ_d'. This is somewhat of a special situation, however, inasmuch as the region of mutual communication intercepts the equator at the same points for these cases. (See fig. 8.) Thus, since $i_{opt} = 0^{\circ}$ in each case, then the results are identical for each case. (See fig. 5.)

The overall effect of increasing station separation distance is closely related to the effect of decreasing the diameter of the region of communication θ_d' inasmuch as both of these conditions lead to a reduction in the size of the region of mutual communication. The main difference in the effects is that the regions are not reduced in quite the same manner; this thus yields different latitude and longitude limits which consequently lead to different optimum orbit inclinations.

Direction of link: In order to determine the effect of the direction of the link, the results for the cases illustrated in figures 4(c), 4(d), or 4(e) can be compared. It would be impractical to analyze all of these curves; therefore, only a typical case is discussed. Consider the links for communicating in all directions from a station at 45° latitude with stations 45° away (cases B-2, R-6, N-1, F-4, R-5, and B-5 in fig. 4(e)). By comparing the results of these cases it is seen that for a given elevation angle and altitude the optimum orbit inclination angle increases from low inclinations (0° to 30°) for a southern link (case B-2) to high inclinations (70° to 90°) for a northern link (case B-5). Also, it should be noted that the maximum value of $P_c(1)$ is greater for links having one station near the equator (case B-2) or North Pole (case B-5). (See results of table III.)

Previously it was mentioned that the curves of figure 5 could be used to determine a compromise value of i and $P_c(1)$ for a set of communication links. For instance, it may be desirable to know the optimum orbit inclination and minimum number of satellites required to communicate in all directions with a common set of satellites. As an example of this, consider again the cases for communicating from a station at 45° latitude with stations 45° away. By comparing the curves of figure 5 for these cases it can be seen that, for $h = 5{,}000$ miles and $\beta = 5^\circ$ (or $\theta_d' = 118^\circ$),

$$i_{opt} \approx 58^\circ$$

$N \approx 33$ for at least 99 percent communicating time in any direction

and, for $h = 3{,}000$ miles and $\beta = 5^\circ$ (or $\theta_d' = 101^\circ$),

$$i_{opt} \approx 55^\circ$$

$N \approx 55$ for at least 99 percent communicating time in any direction

<u>Sample worldwide communications system</u>.- Results for a sample worldwide communications system are presented in order to illustrate how the curves for the systematic study can be utilized. The communication links for this system were chosen primarily on the basis of yielding intercontinental communications and also for communications between large cities. A list of the communication links considered is given in table IV. These links are illustrated on a map in figure 9. Notice that a large number of North Atlantic links was studied. This study was made because these links will probably be utilized heavily in the first and any subsequent commercial applications of communication satellites. Only one angular diameter of the region of communication for each station is considered in this sample study, namely, $\theta_d' = 118^\circ$ (for example, $h = 5{,}000$ miles and $\beta = 5^\circ$). (See fig. 3 for other combinations of h and β to which this value applies.)

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The results of this study in the form of plots of $P_c(1)$ as a function of i are given in figure 10. By comparing these curves it is seen that the optimum orbit inclination angle for this worldwide system is about 79° with a minimum value of $P_c(1)$ of 0.076, which would require 59 satellites to communicate at least 99 percent of the time over any one of these links. (See fig. 6.) However, by allowing a slightly lower percentage of communication time on some of the links, the required number of satellites could be reduced considerably. For example, by reducing the percentage of communication time on the links shown in table V only to the values indicated therein, it was found that 45 satellites in orbits inclined 80° to the equator would allow communication (a) over 99.9 percent of the time over 51 percent of the links, (b) 99.9 to 99.0 percent of the time over an additional 33 percent of the links, and (c) 99.0 to 97.0 percent of the time over the remaining 16 percent of the links. A total of 70 links was investigated. This worldwide system is presented merely as one example of what can be done with the curves presented in this systematic study. Many modifications can be made depending on which links are considered to be the most important.

Systematic Study Based on Circular Region

The results of the study based on a circular region of mutual communication are presented in figure 11 in the form of plots showing the variation of the probability of communicating having a single satellite in orbit $P_c(1)$ with orbit inclination angle i. As in the study based on a lenticular region, these curves apply for combinations of h and β other than the particular ones given in figure 11. (See fig. 3.) The procedure for determining the required number of satellites is the same as in the preceding study.

The parameters of interest in this study are h, β, and station latitude (or latitude of the center of the circle). The effects of these variables on the curves of $P_c(1)$ as a function of i are similar to those for the preceding study.

For a practical worldwide navigation (or communication) satellite system it is desirable to know the required number of satellites for communicating from all latitudes with a common set of satellites. In order to do this, the results of figure 11 for each latitude and for a given circle diameter (or h and β combination) must be superimposed to first obtain compromise values of i and $P_c(1)$. This procedure is illustrated in figure 12 for a circle diameter of 96° (h = 2,000 miles and $\beta = 0^{\circ}$). The compromise inclination angle is seen to be about 54°. The minimum value of $P_c(1)$ at this value of i is 0.157 for which the minimum number of satellites required for communicating at least 99 percent of the time at all latitudes is 27. (See fig. 6.) A similar analysis is made for other circle diameters and the results are given in table VI. From this table it can be seen that the optimum value of i for communicating on a worldwide basis, that is, from all latitudes, based on a circular region of mutual communication, is between 53° and 64° for all of the circle diameters considered.

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CONCLUDING REMARKS

Certain geometric aspects of the communication and navigation satellite concepts have been presented. The study was limited to satellites deployed singly in randomly spaced, near-earth, circular orbits.

Results have been presented of two systematic studies from which estimates of the required number of satellites for many practical communication and navigation links can be obtained. One study was based on a lenticular region of mutual communication, and the other was based on a circular region.

Results for a sample worldwide communications system based on the lenticular region were also presented. The communication links for this system were chosen primarily for intercontinental and large city communications. A total of 70 links was studied. At an altitude where the angular diameter of the circular region of communication at each station is 118°, it was found that 45 satellites in orbits inclined 80° to the equator would allow communication (a) over 99.9 percent of the time over 51 percent of the links, (b) 99.9 to 99.0 percent of the time over an additional 33 percent of the links, and (c) 99.0 to 97.0 percent of the time over the remaining 16 percent of the links.

For a worldwide navigation satellite system (or communication system based on circular regions of mutual communication) it was found that, for the range of circle diameters considered, the optimum orbit inclination angle lies between 53° and 64°.

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National Aeronautics and Space Administration,
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TABLE I

STATION LOCATIONS FOR SYSTEMATIC STUDY BASED ON LENTICULAR REGION OF MUTUAL COMMUNICATION

Case	Latitude of station 1, deg	Latitude of station 2, deg	$\Delta\lambda$, deg	Station separation distance
Links inclined 90° (North-South)				
A-1	-15	15	0	30
A-2	0	30	0	30
A-3	15	45	0	30
A-4	30	60	0	30
A-5	45	75	0	30
A-6	60	90	0	30
A-7	75	75	180	30
B-1	-15	30	0	45
B-2	0	45	0	45
B-3	15	60	0	45
B-4	30	75	0	45
B-5	45	90	0	45
B-6	60	75	180	45
C-1	-30	30	0	60
C-2	-15	45	0	60
C-3	0	60	0	60
C-4	15	75	0	60
C-5	30	90	0	60
C-6	45	75	180	60
C-7	60	60	180	60
D-1	-30	45	0	75
D-2	-15	60	0	75
D-3	0	75	0	75
D-4	15	90	0	75
D-5	30	75	180	75
D-6	45	60	180	75
Links inclined 0° (East-West)				
E-1	0	0	30.0	30
E-2	15	15	31.1	30
E-3	30	30	34.8	30
E-4	45	45	42.9	30
E-5	60	60	62.3	30
*E-6	75	75	180.0	30
F-1	0	0	45.0	45
F-2	15	15	46.7	45
F-3	30	30	52.4	45
F-4	45	45	65.5	45
F-5	60	60	99.9	45
G-1	0	0	60.0	60
G-2	15	15	62.3	60
G-3	30	30	70.5	60
G-4	45	45	90.0	60
**G-5	60	60	180.0	60
H-1	0	0	75.0	75
H-2	15	15	78.1	75
H-3	30	30	89.3	75
H-4	45	45	118.8	75

*Same as case A-7.
**Same as case C-7.

Case	Latitude of station 1, deg	Latitude of station 2, deg	$\Delta\lambda$, deg	Station separation distance
Links inclined 30°				
I-1	15	0.6	26.6	30
I-2	15	26.0	29.9	30
I-3	30	25.7	33.7	30
J-1	15	-6.9	39.7	45
J-2	15	29.1	46.5	45
J-3	30	20.7	49.1	45
K-1	30	14.5	63.4	60
K-2	15	-14.0	53.1	60
K-3	15	30.0	63.7	60
L-1	30	7.4	76.9	75
L-2	15	-20.3	67.4	75
L-3	15	28.8	80.9	75
Links inclined 45°				
M-1	45	37.8	39.3	30
N-1	45	30.0	54.7	45
O-1	45	20.7	67.4	60
P-1	45	10.6	79.2	75
Links inclined 60°				
Q-1	15	10.9	15.3	30
Q-2	15	39.6	19.6	30
Q-3	30	51.9	27.9	30
Q-4	30	4.6	16.9	30
Q-5	45	59.6	44.3	30
Q-6	45	21.4	22.8	30
R-1	15	23.7	23.6	45
R-2	15	50.1	34.8	45
R-3	30	58.6	51.6	45
R-4	30	-8.4	24.3	45
R-5	30	58.6	73.7	45
R-6	45	8.6	30.7	45
S-1	30	21.2	32.4	60
S-2	15	35.9	33.6	60
S-3	15	57.7	57.0	60
S-4	30	59.6	81.0	60
S-5	45	51.9	97.4	60
S-6	45	-4.4	38.3	60
T-1	30	33.6	42.0	75
T-2	15	47.0	47.1	75
T-3	15	59.9	85.9	75
T-4	30	54.3	107.0	75
T-5	45	41.8	113.7	75
T-6	45	-17.3	46.0	75

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TABLE II

SAMPLE RESULTS SHOWING EFFECT OF STATION SEPARATION DISTANCE

(a) Southern links

θ_d', deg	h, miles	β, deg	Case A-3			Case B-2			Case C-2			Case D-1		
			i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$
138	5,000	0	0	0.285	14	0	0.285	14	0	0.285	14	0	0.285	14
109	5,000	10	0	.190	22	0	.190	22	0	.190	22	0	.190	22
87	2,000	5	40	.105	42	27	.089	50	17	.071	64	10	.047	96

(b) Northern links

θ_d', deg	h, miles	β, deg	Case A-5			Case B-5			Case C-6			Case D-6		
			i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$
138	5,000	0	90	0.277	15	90	0.249	16	90	0.224	18	90	0.193	22
109	5,000	10	90	.211	20	90	.185	23	90	.160	26	90	.128	34
87	2,000	5	85	.120	36	85	.097	46	85	.071	63	85	.038	120

(c) East-West links

θ_d', deg	h, miles	β, deg	Case E-4			Case F-4			Case G-4			Case H-4		
			i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$	i_{opt}, deg	$P_c(1)$	$N_{0.99}$
138	5,000	0	90	0.266	15	90	0.238	17	90	0.208	20	90	0.181	23
109	5,000	10	90	.196	21	90	.168	25	90	.140	31	90	.111	39
87	2,000	5	82	.101	44	82	.093	47	82	.045	100	82	.019	245

TABLE III

SAMPLE RESULTS SHOWING VARIATION OF $\left[P_c(1)\right]_{max}$ L-1858

WITH DIRECTION OF LINK

$\left[h = 3{,}000 \text{ miles}, \quad \beta = 5^{\circ}\right]$

Case	Inclination of link, deg	i_{opt}, deg	$\left[P_c(1)\right]_{max}$
B-2	90	0	0.143
R-6	60	0	.125
N-1	45	75	.102
F-4	0	90	.138
R-5	60	90	.152
B-5	90	90	.157

L-1858

TABLE IV

STATION LOCATIONS FOR SAMPLE WORLDWIDE COMMUNICATIONS SYSTEM

Link	Station 1			Station 2		
	Location	Latitude, deg	Longitude, deg	Location	Latitude, deg	Longitude, deg
			North Atlantic links			
1	New York	41	74	London	52	0
2	New York	41	74	Paris	49	-2
3	New York	41	74	Bonn	51	-10
4	New York	41	74	Berlin	53	-13
5	New York	41	74	Lisbon	39	9
6	New York	41	74	Madrid	41	4
7	New York	41	74	Casablanca	33	7
8	Ottawa	45	76	London	52	0
9	Newfoundland	52	56	Ireland	52	10
10	New York	41	74	Rome	42	-13
11	Miami	26	80	Dakar, French West Africa	15	17
12	Norfolk	37	76	Casablanca	33	7
13	Washington	39	77	Moscow	56	-38
			European and African links			
1	Casablanca	33	7	Ankara	40	-33
2	Lisbon	39	9	Cairo	30	-31
3	Lisbon	39	9	Moscow	56	-38
4	London	52	0	Moscow	56	-38
5	London	52	0	Ankara	40	-33
6	Berlin	53	-13	Ankara	40	-33
7	Dakar, French West Africa	15	17	Addis Ababa, Ethiopia	9	-39
			Eurasian and African-Asian links			
1	Berlin	53	-13	New Delhi	28	-77
2	Ankara	40	-33	New Delhi	28	-77
3	Cairo	30	-31	Bombay	19	-73
4	Ankara	40	-33	Bombay	19	-73
5	Moscow	56	-38	New Delhi	28	-77
6	Cairo	30	-31	New Delhi	28	-77
7	Rome	42	-3	New Delhi	28	-77
8	Addis Ababa, Ethiopia	9	-39	Bombay	19	-73
9	Addis Ababa, Ethiopia	9	-39	Ceylon	7	-80
			Asian-Far East Pacific links			
1	New Delhi	28	-77	Tokyo	36	-139
2	New Delhi	28	-77	Manila	14	-121
3	Bombay	19	-73	Tokyo	36	-139
4	Bombay	19	-73	Manila	14	-121
5	Ceylon	7	-80	Manila	14	-121
			Pacific links			
1	Tokyo	36	-139	Los Angeles	34	118
2	Tokyo	36	-139	Hawaii	20	155
3	Tokyo	36	-139	Wake Island	19	-167
4	Tokyo	36	-139	Manila	14	-121
5	Tokyo	36	-139	Darwin	-13	-131
6	Manila	14	-121	Darwin	-13	-131
7	Manila	14	-121	Wake Island	19	-167
8	Tokyo	36	-139	Anchorage	61	150
9	Sydney	-34	-151	South Pole	-90	----
10	Sydney	-34	-151	Oates Coast, Antarctica	-67	-141
11	Sydney	-34	-151	Fiji Islands	-14	180
12	Fiji Islands	-14	180	Wake Island	19	-167
13	Fiji Islands	-14	180	Hawaii	20	155
14	Wake Island	19	-167	Anchorage	61	150
15	Wake Island	19	-167	Hawaii	20	155
16	Hawaii	20	155	Anchorage	61	150
17	Anchorage	61	150	Seattle	48	122
18	Anchorage	61	150	Los Angeles	34	118
19	Hawaii	20	155	Los Angeles	34	118
			North and South American links			
1	Los Angeles	34	118	New York	41	74
2	Miami	26	80	Panama	9	79
3	Miami	26	80	Lima, Peru	-12	77
4	Miami	26	80	Natal, Brazil	-6	35
5	Panama	9	79	Lima, Peru	-12	77
6	Punta Arenas, Argentina	-53	71	Lima, Peru	-12	77
7	Punta Arenas, Argentina	-53	71	Palmer Peninsula, Antarctica	-67	66
8	Punta Arenas, Argentina	-53	71	South Pole	-90	----
9	Newfoundland	52	56	Thule, Greenland	76	68
10	Newfoundland	52	56	North Pole	90	----
11	Seattle	48	122	North Pole	90	----
			South Atlantic and Indian Ocean links			
1	Natal, Brazil	-6	35	Dakar, French West Africa	15	17
2	Natal, Brazil	-6	35	Windhoek, Southwest Africa	-23	-17
3	Natal, Brazil	-6	35	Monrovia, Liberia	6	11
4	Rio de Janeiro	-23	44	Windhoek, Southwest Africa	-23	-17
5	Montevideo, Uruguay	-35	56	Windhoek, Southwest Africa	-23	-17
6	Johannesburg	-26	-28	Perth, Australia	-32	-116

TABLE V

LINKS FOR WORLDWIDE SYSTEM WITH COMMUNICATION TIME LESS THAN

99.0 PERCENT FOR 45 SATELLITES AND $i = 80^\circ$

L
1
8
5
8

Link	$P_c(N)$
Miami-Dakar	0.982
Dakar-Addis Ababa	0.982
Ankara-Bombay	0.978
Bombay-Tokyo	0.983
Bombay-Manila	0.989
Ceylon-Manila	0.989
Tokyo-Los Angeles	0.973
Tokyo-Darwin	0.983
Miami-Natal	0.978
Johannesburg-Perth	0.972
Natal-Windhoek	0.984
Natal-Monrovia	0.986

TABLE VI

RESULTS OF SYSTEMATIC STUDY BASED ON CIRCULAR REGION FOR COMMUNICATING AT ALL LATITUDES SIMULTANEOUSLY

h, miles	β, deg	θ_d', deg	$\left[P_c(1)\right]_{min}$	i, deg	$N_{0.99}$
1,000	0	74	0.084	56	52
2,000	0	96	.157	54	27
3,000	0	110	.210	56	20
5,000	0	128	.265	63	15
1,000	5	65	.060	60	75
2,000	5	87	.122	55	36
3,000	5	101	.177	53	24
5,000	5	118	.232	59	18
1,000	10	56	.044	64	102
2,000	10	78	.095	56	27
3,000	10	92	.139	54	31
5,000	10	109	.201	55	21

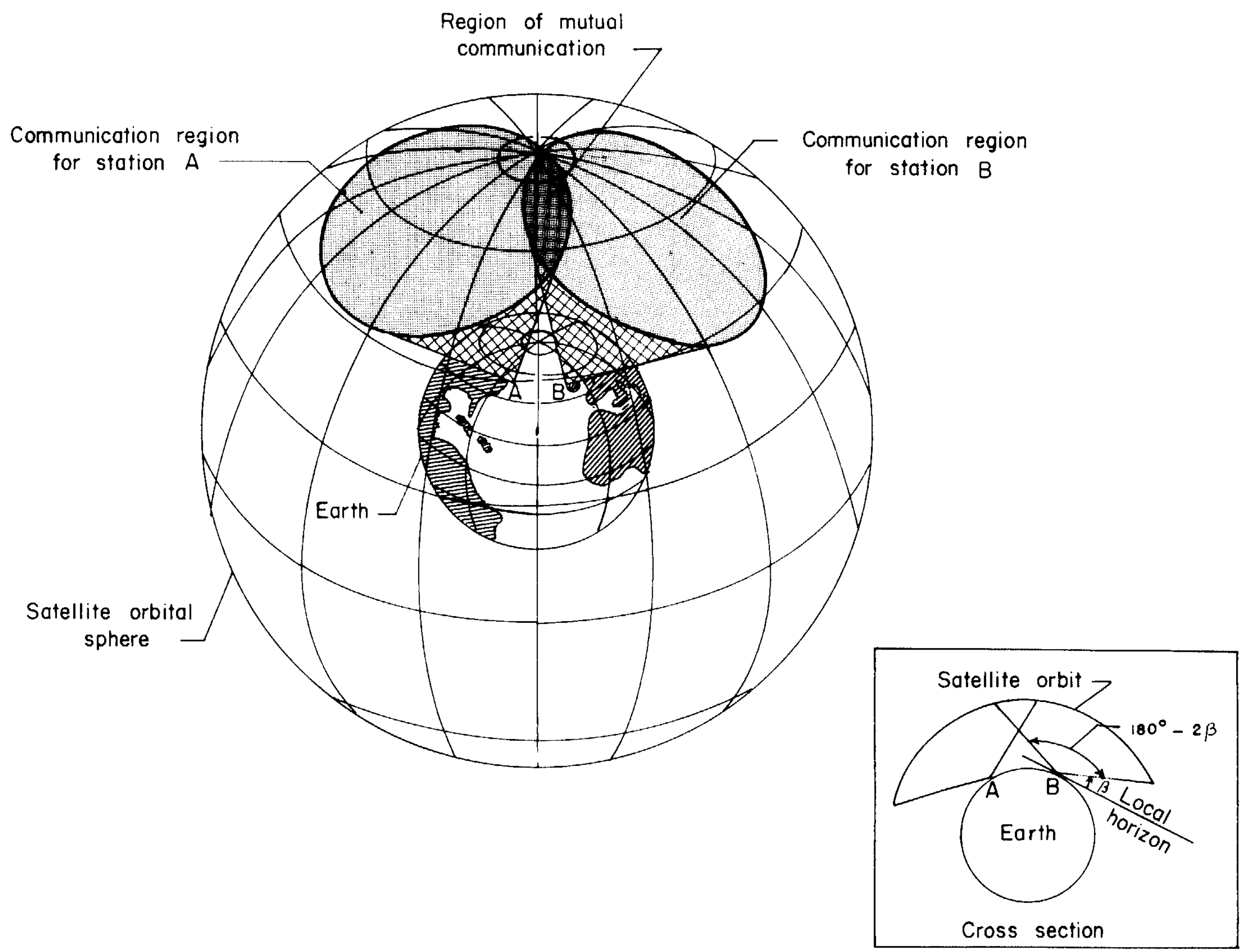

Figure 1.- Problem geometry.

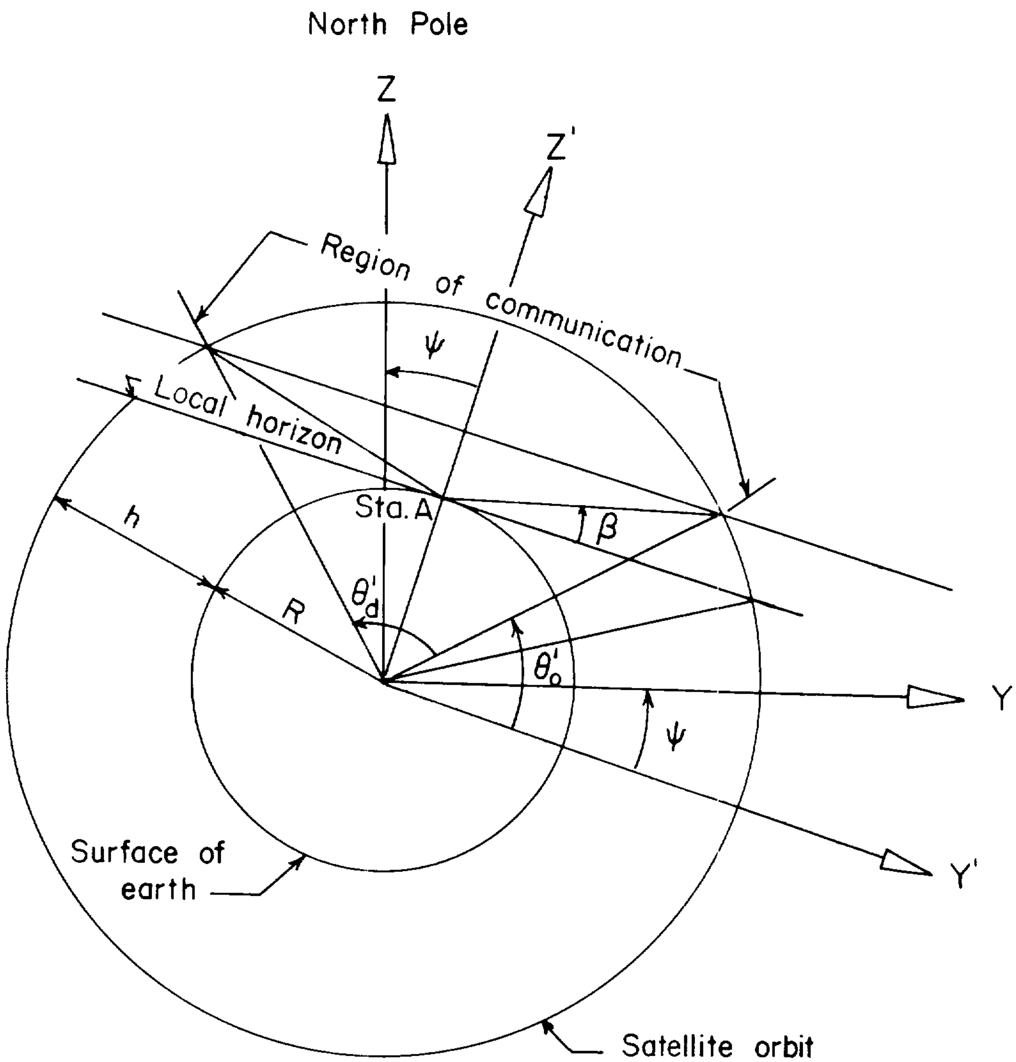

Figure 2.- Cross section of region of communication.

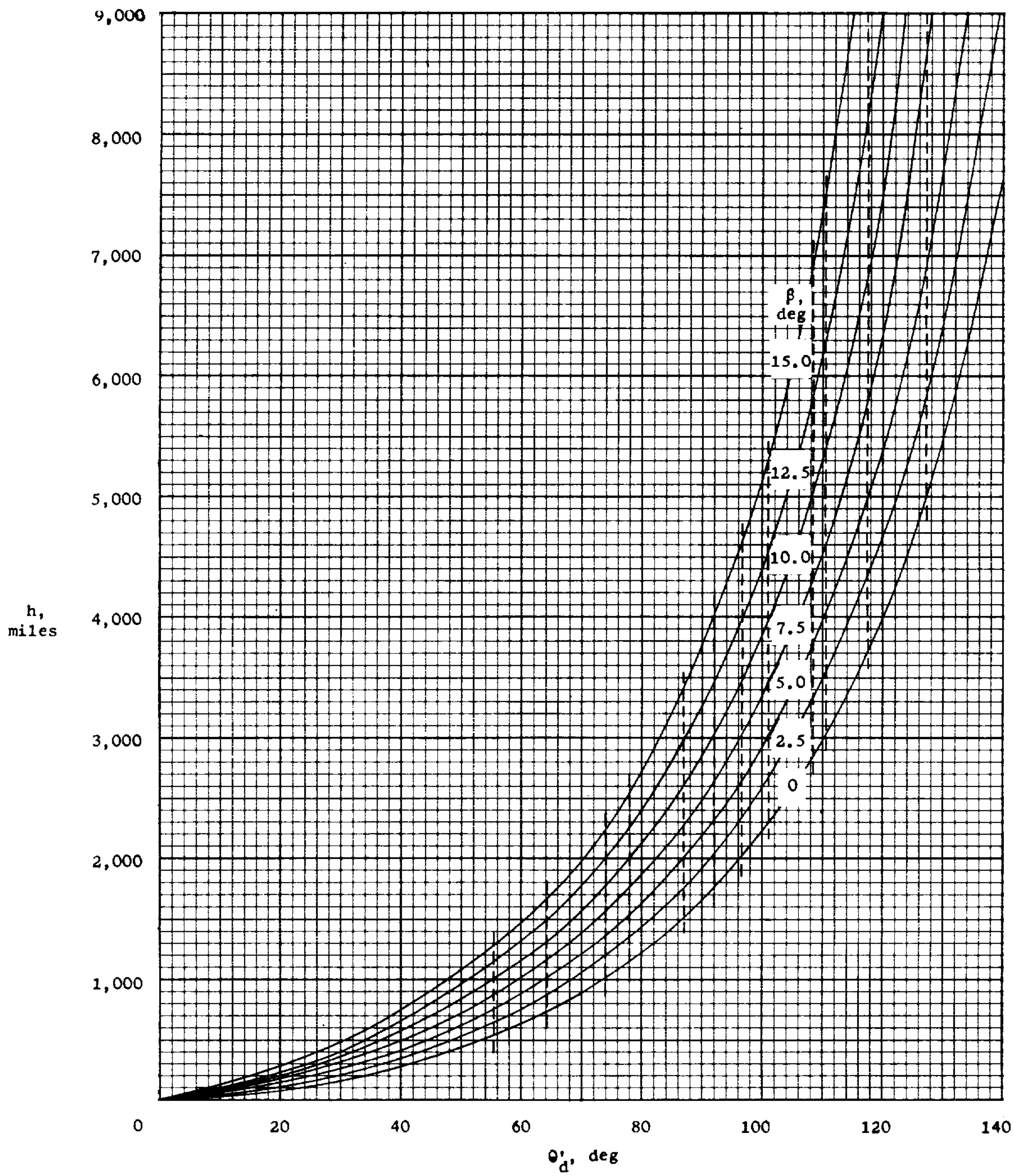

Figure 3.- Combinations of h and β that fix θ_d'.

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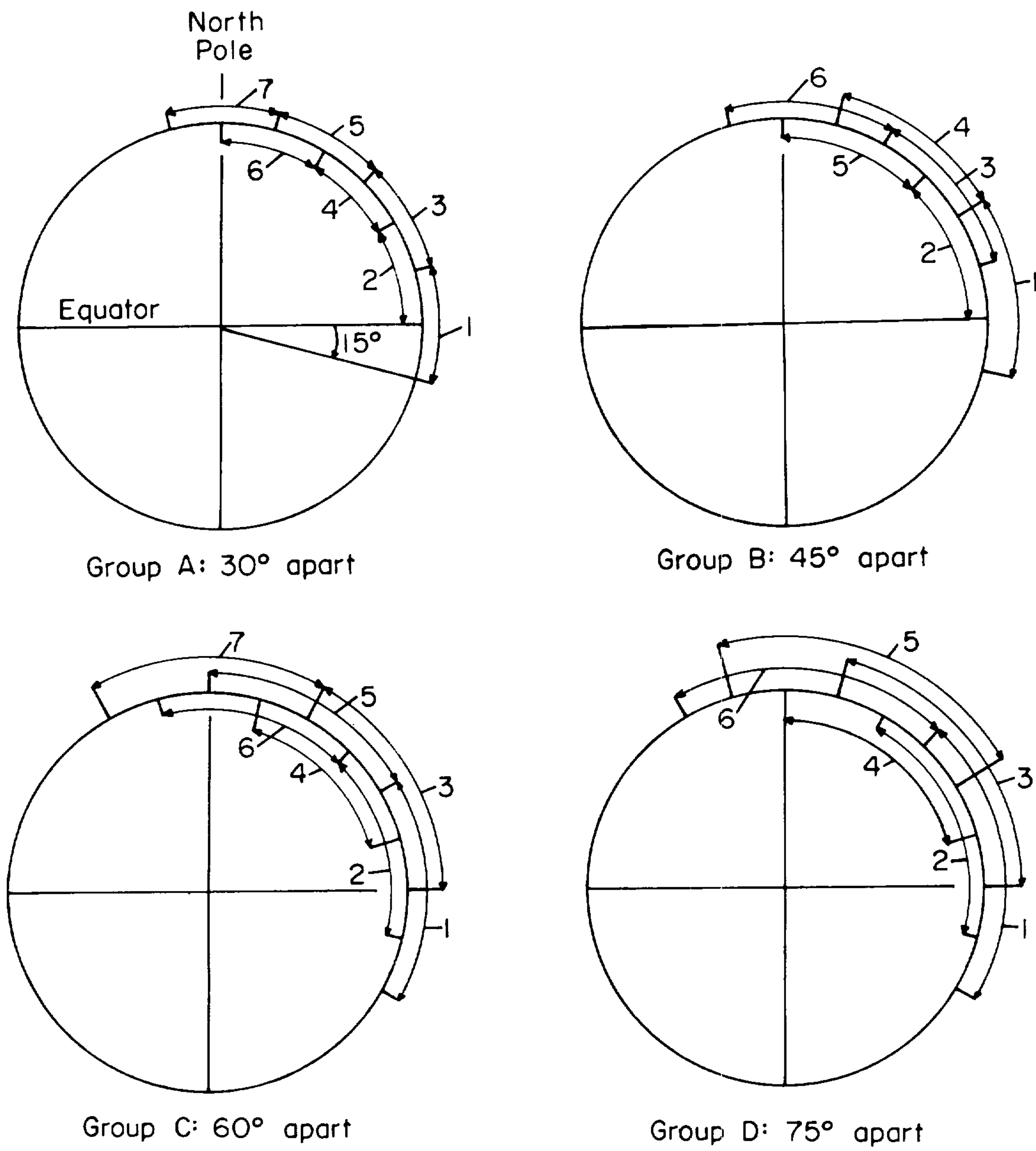
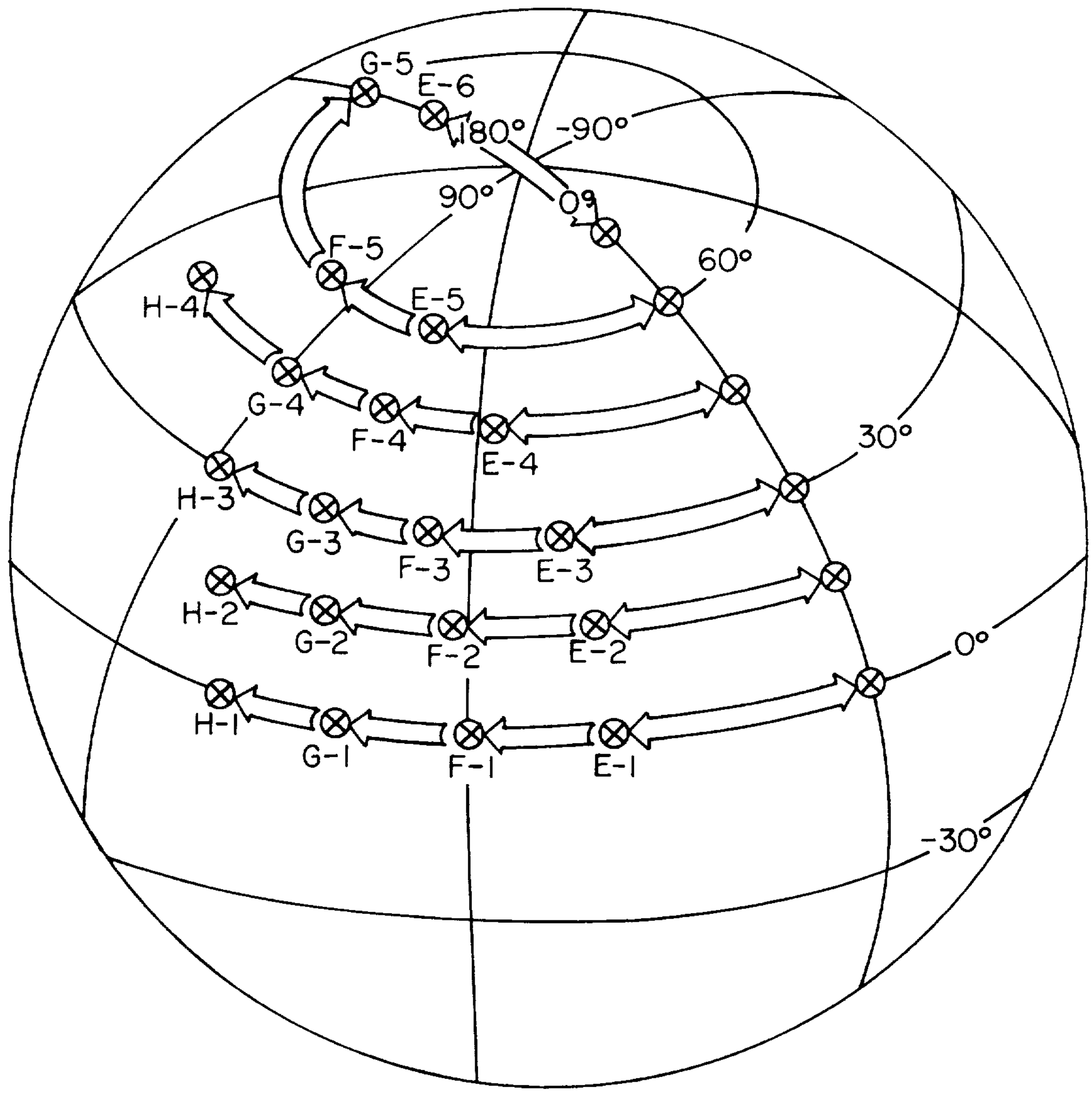


(a) North-south links.

Figure 4.- Sketches showing station locations for systematic lenticular study.



(b) East-west links.

Figure 4.- Continued.

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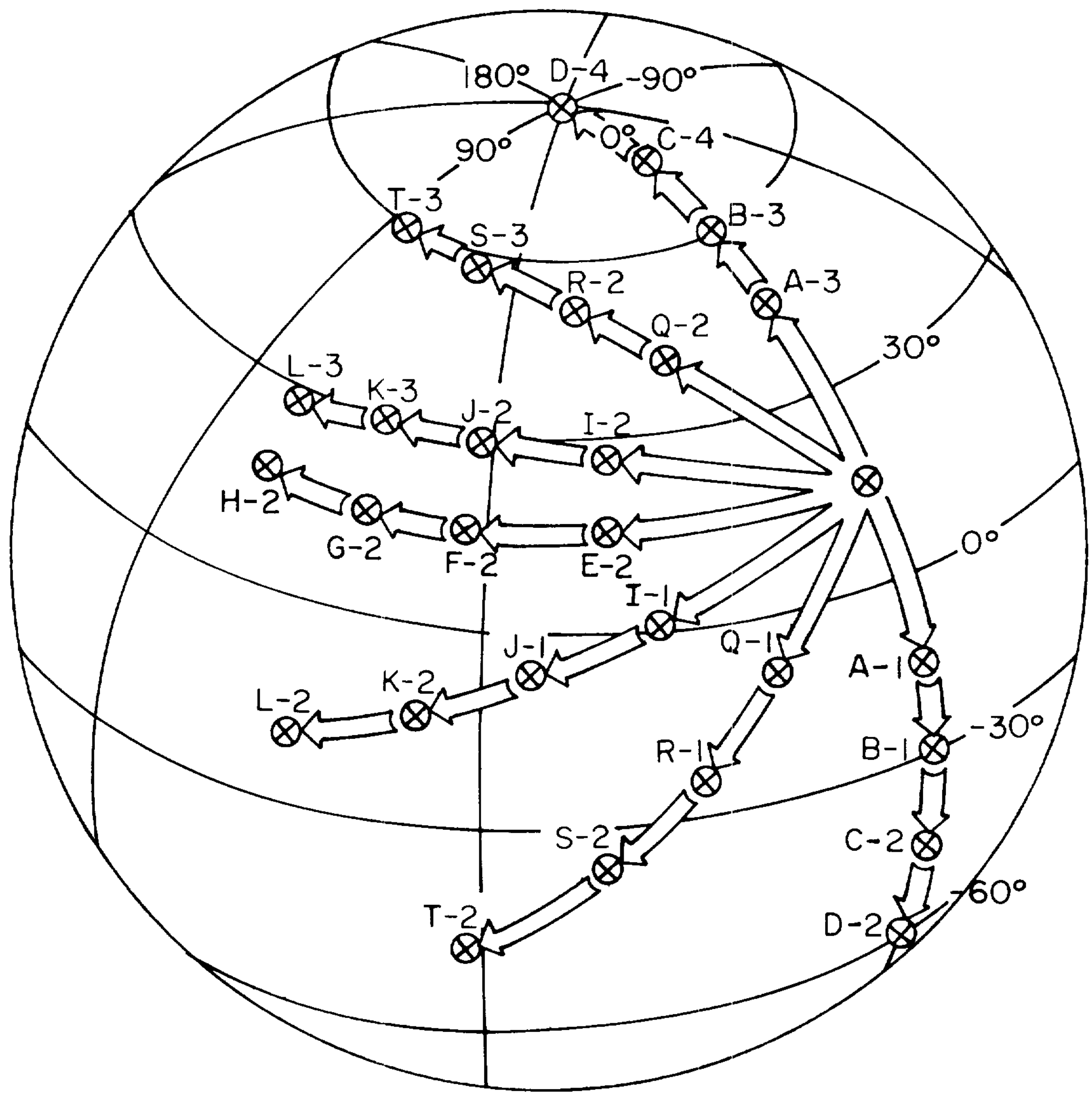
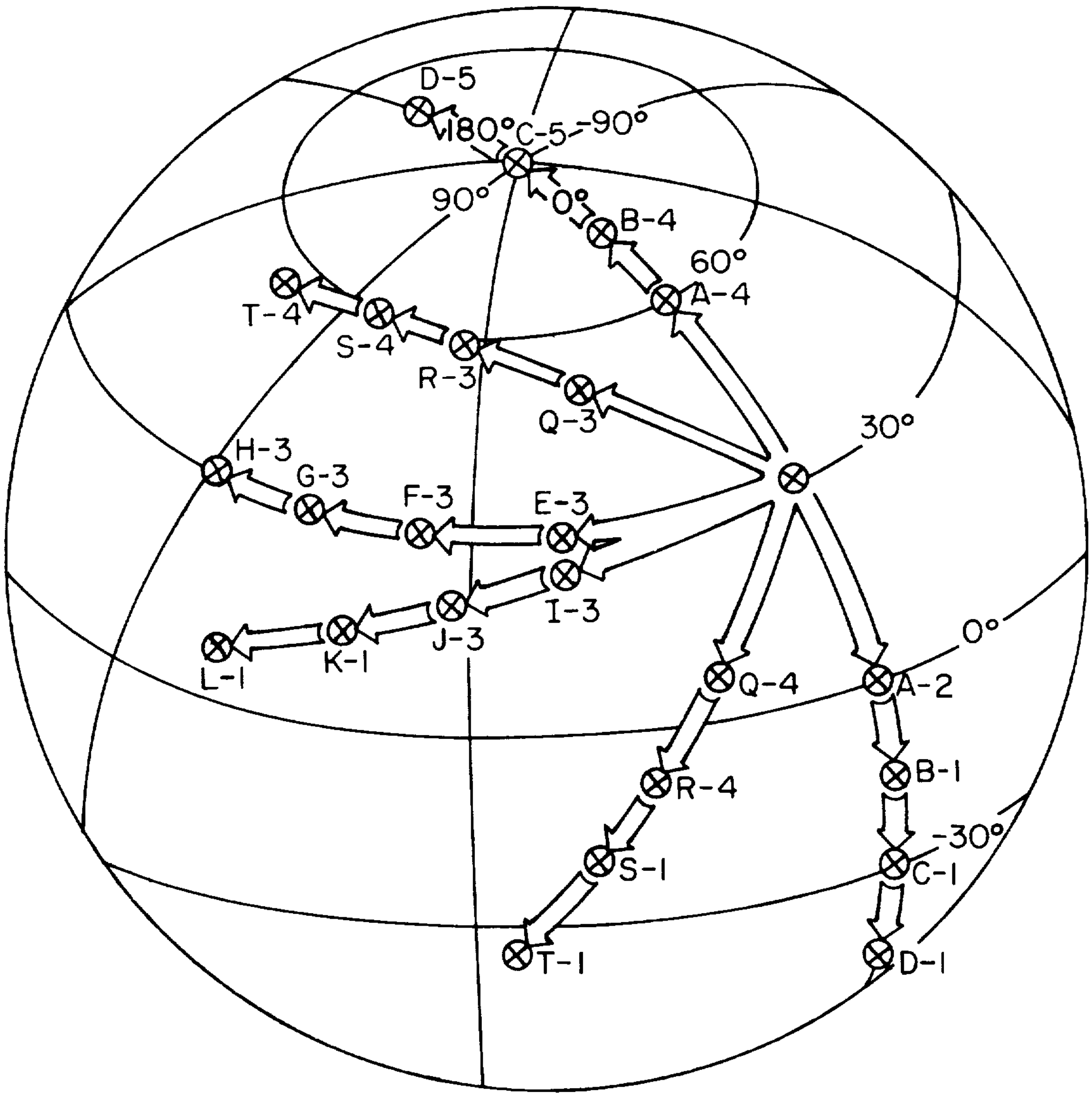


(c) Multidirectional links from 15° latitude.

Figure 4.- Continued.



(d) Multidirectional links from 30° latitude.

Figure 4.- Continued.

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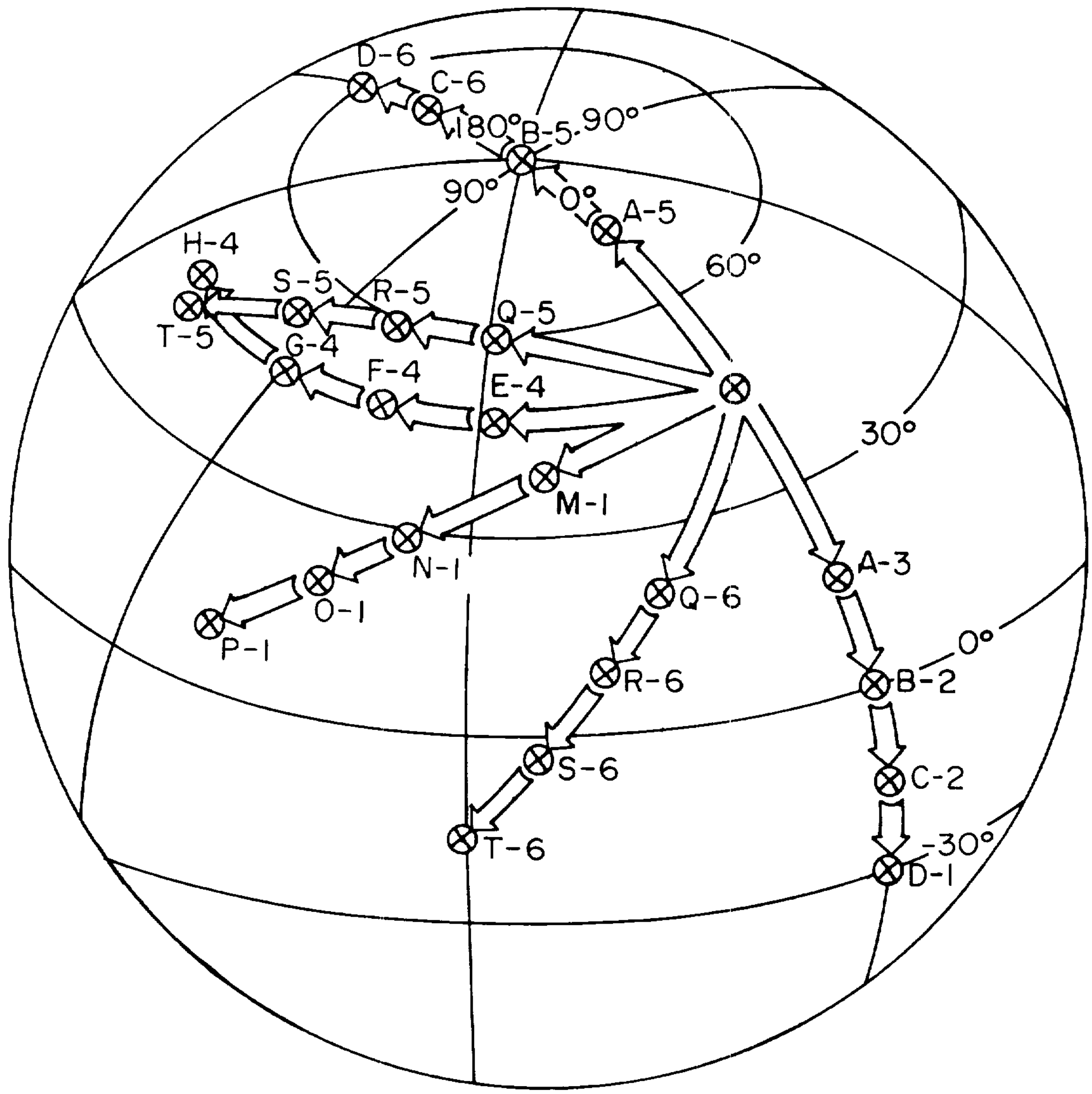

(e) Multidirectional links from 45° latitude.

Figure 4.- Concluded.

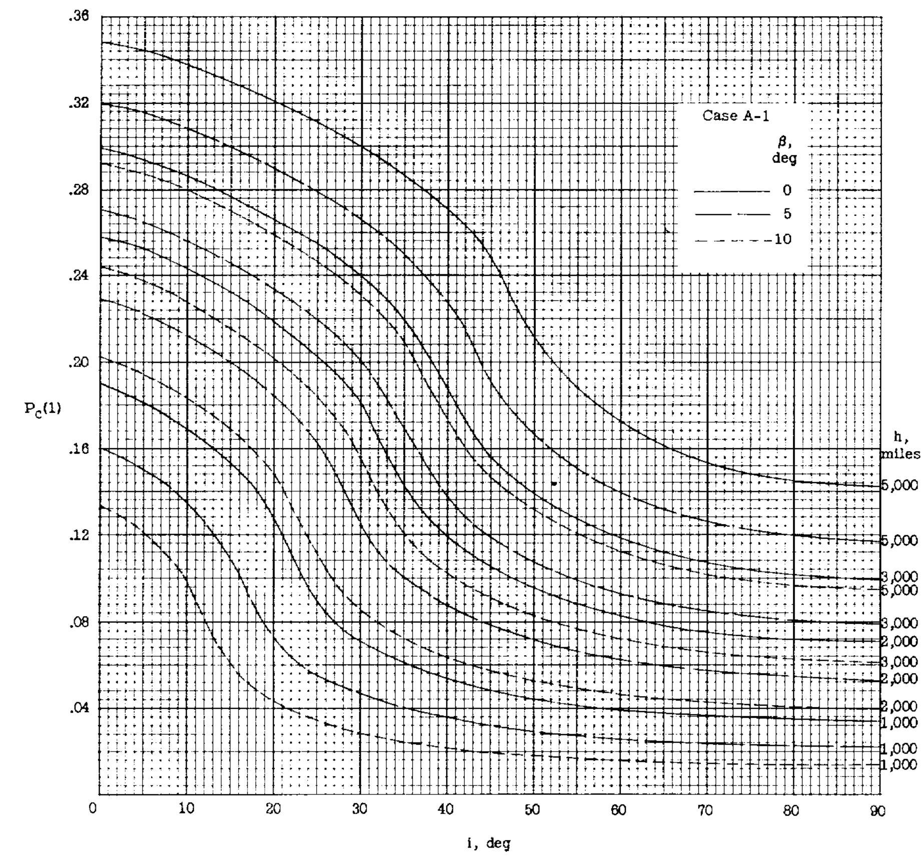

Figure 5.- Variation of probability of communicating having only one satellite in orbit with orbit inclination angle for systematic lenticular study. (Note: These curves apply for combinations of h and β other than the particular ones listed in the figure. See fig. 3.)

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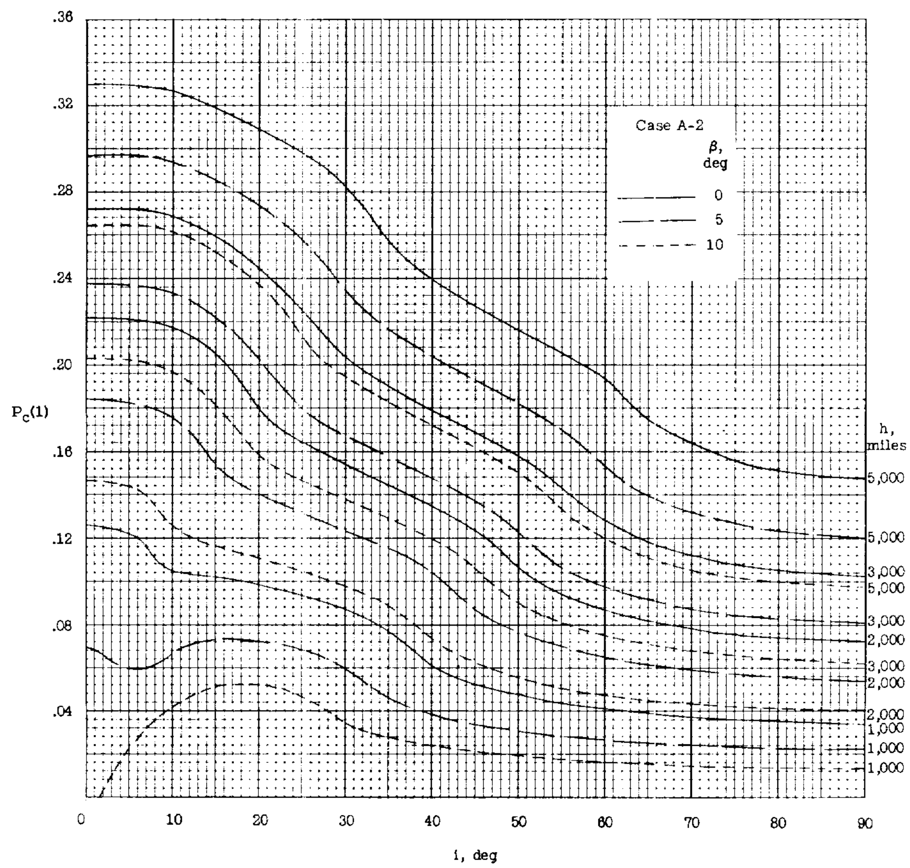

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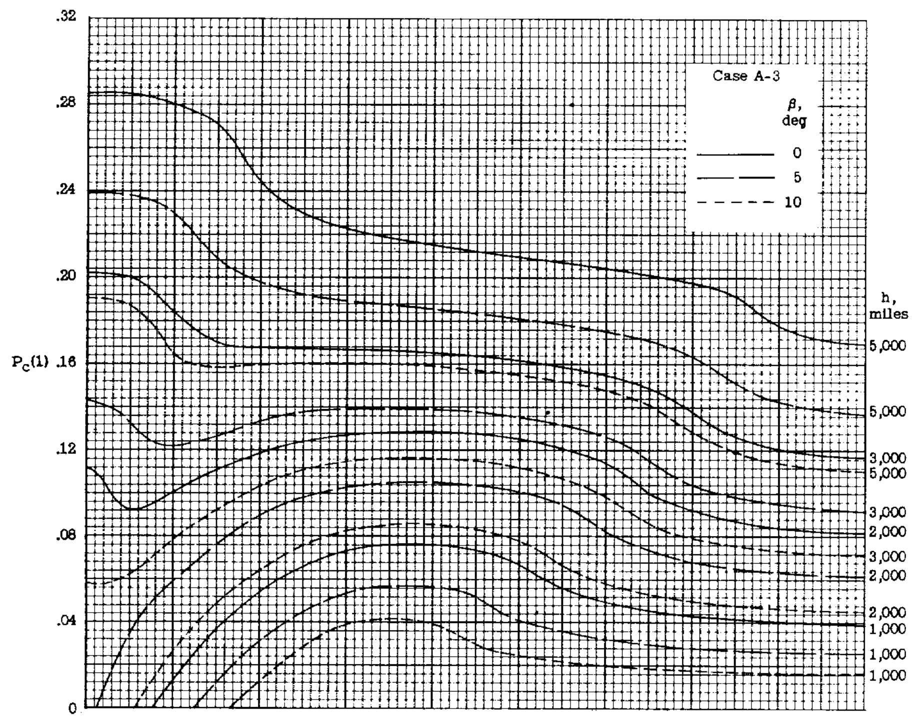

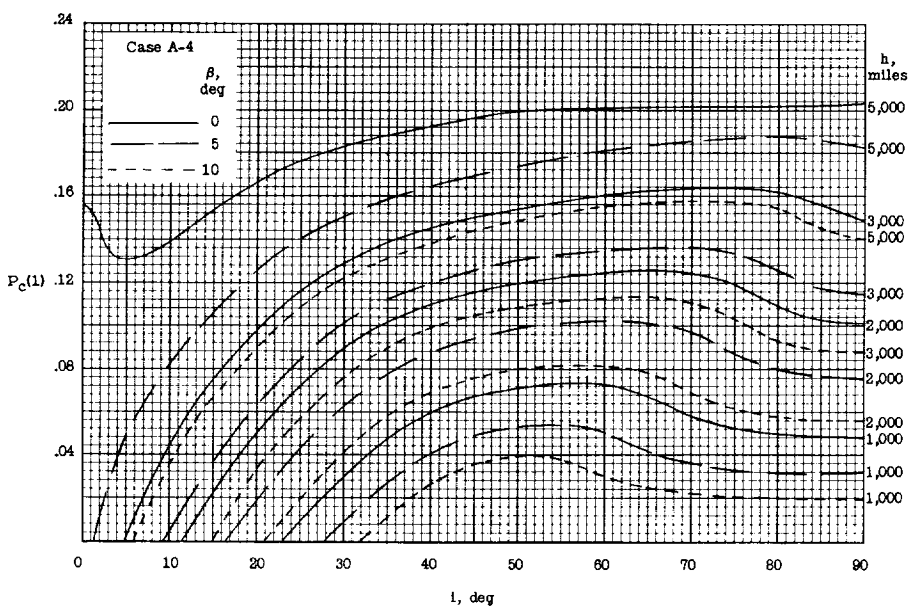

Figure 5.- Continued.

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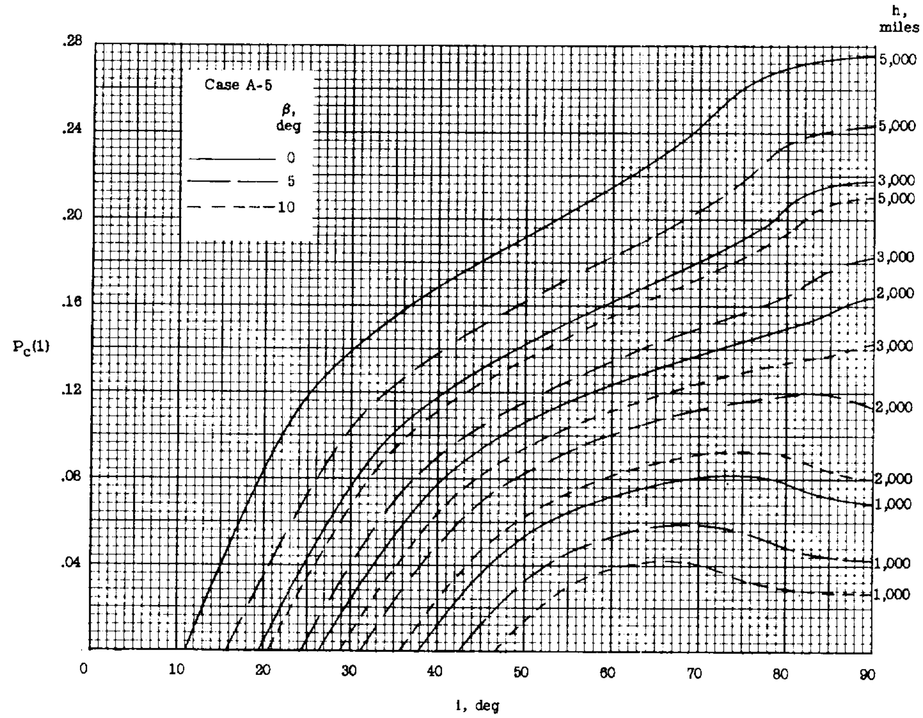

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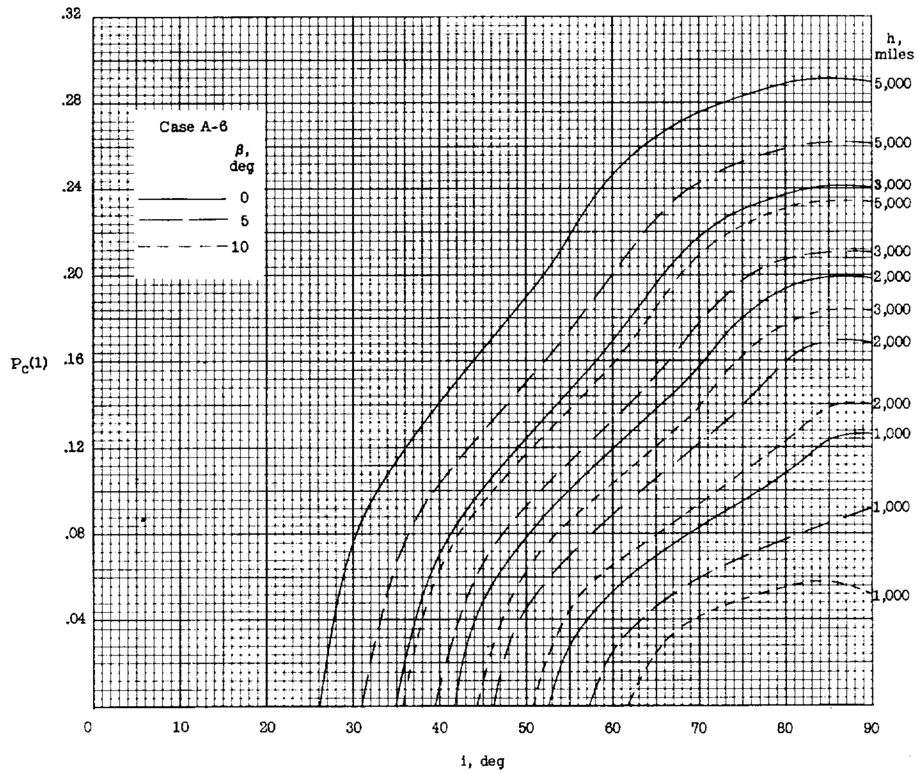

Figure 5.- Continued.

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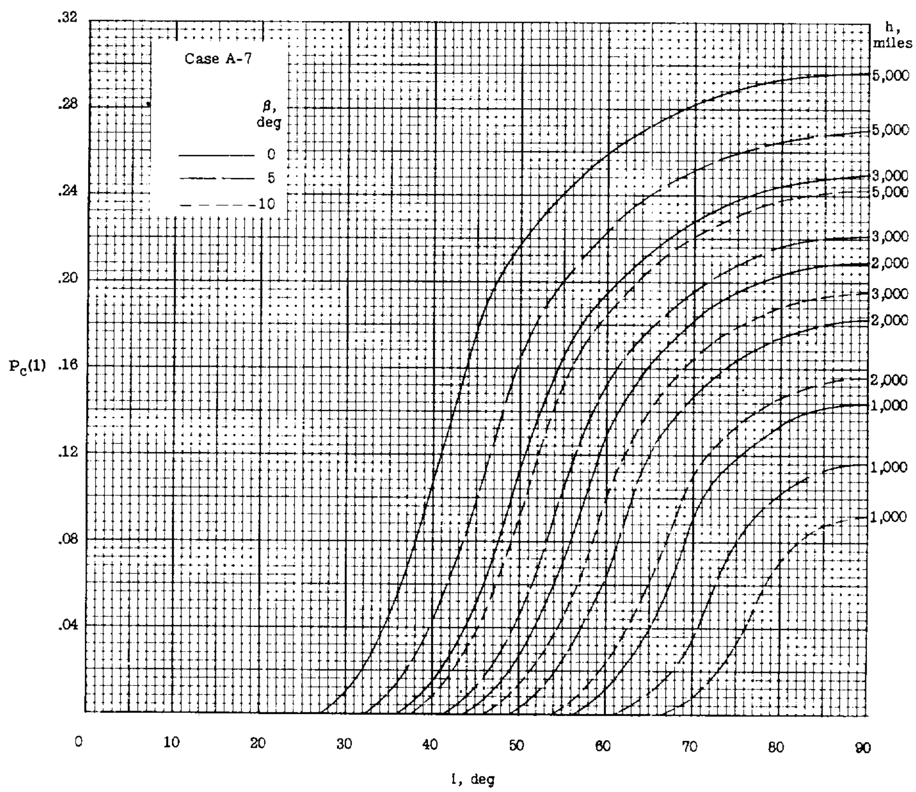

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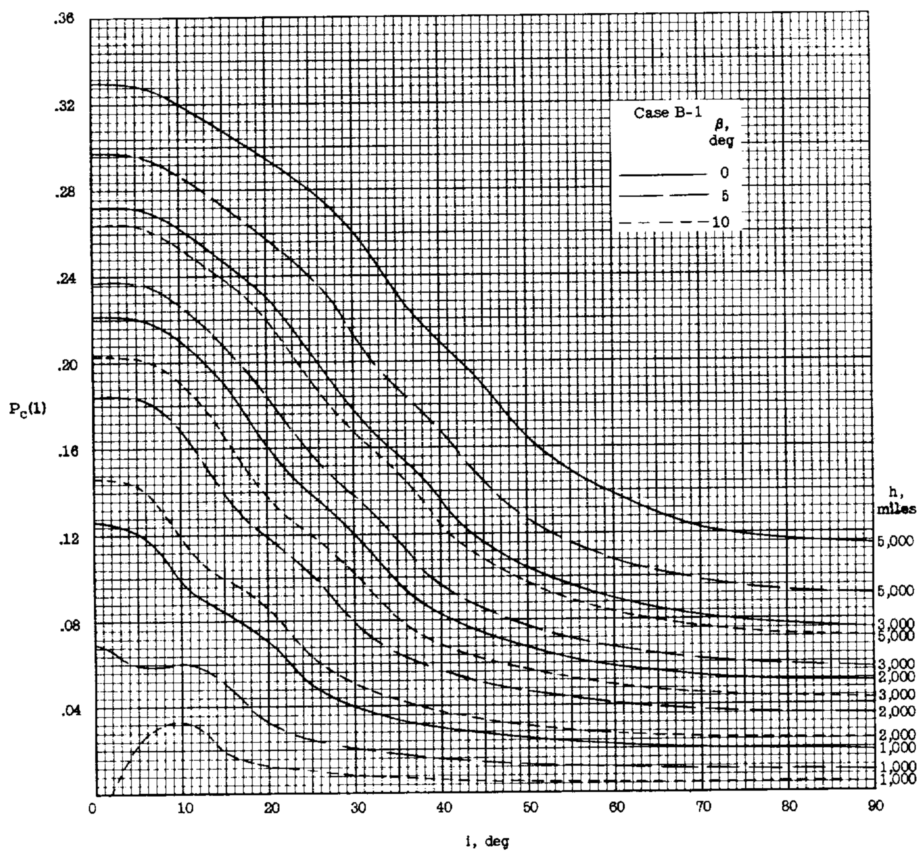

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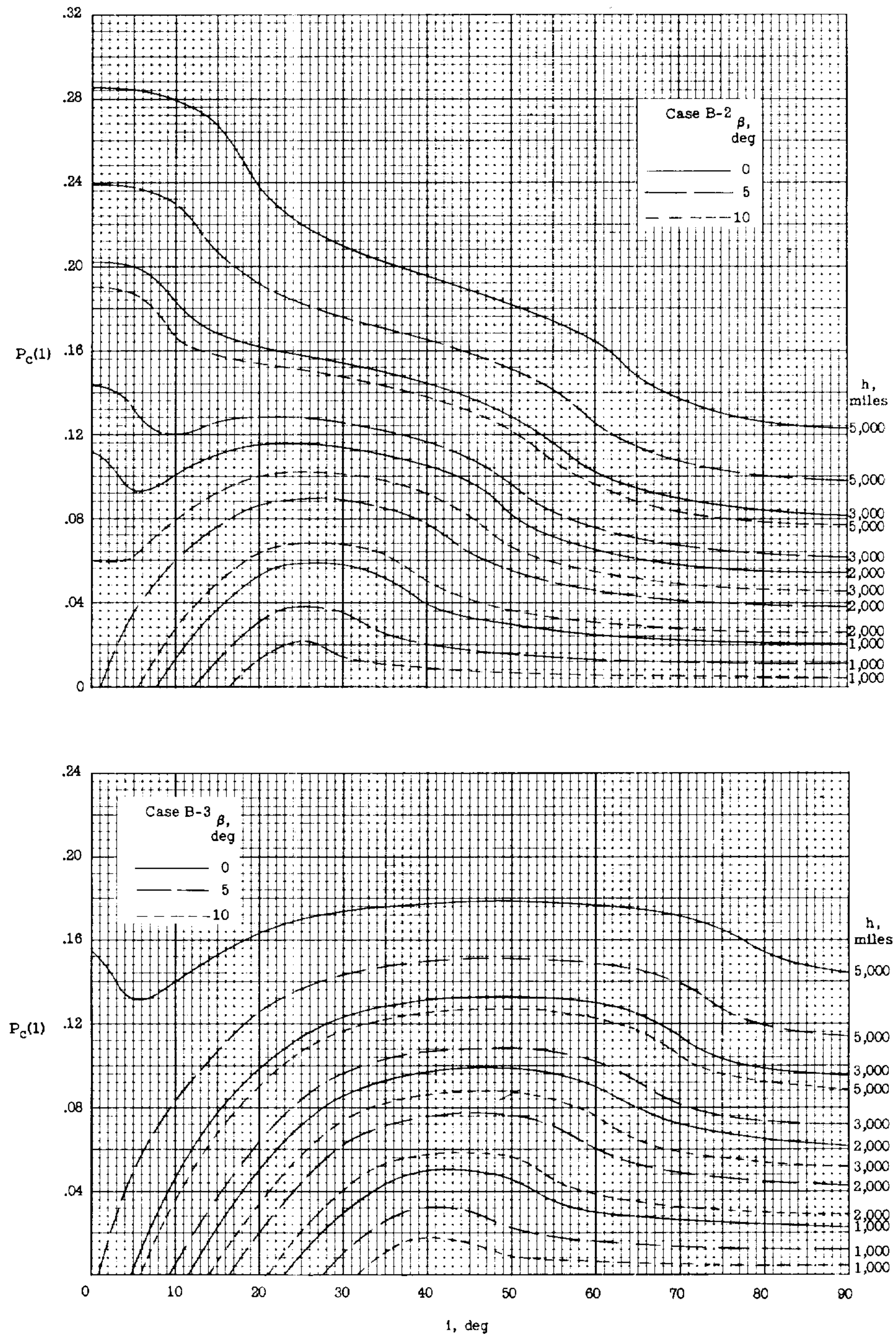

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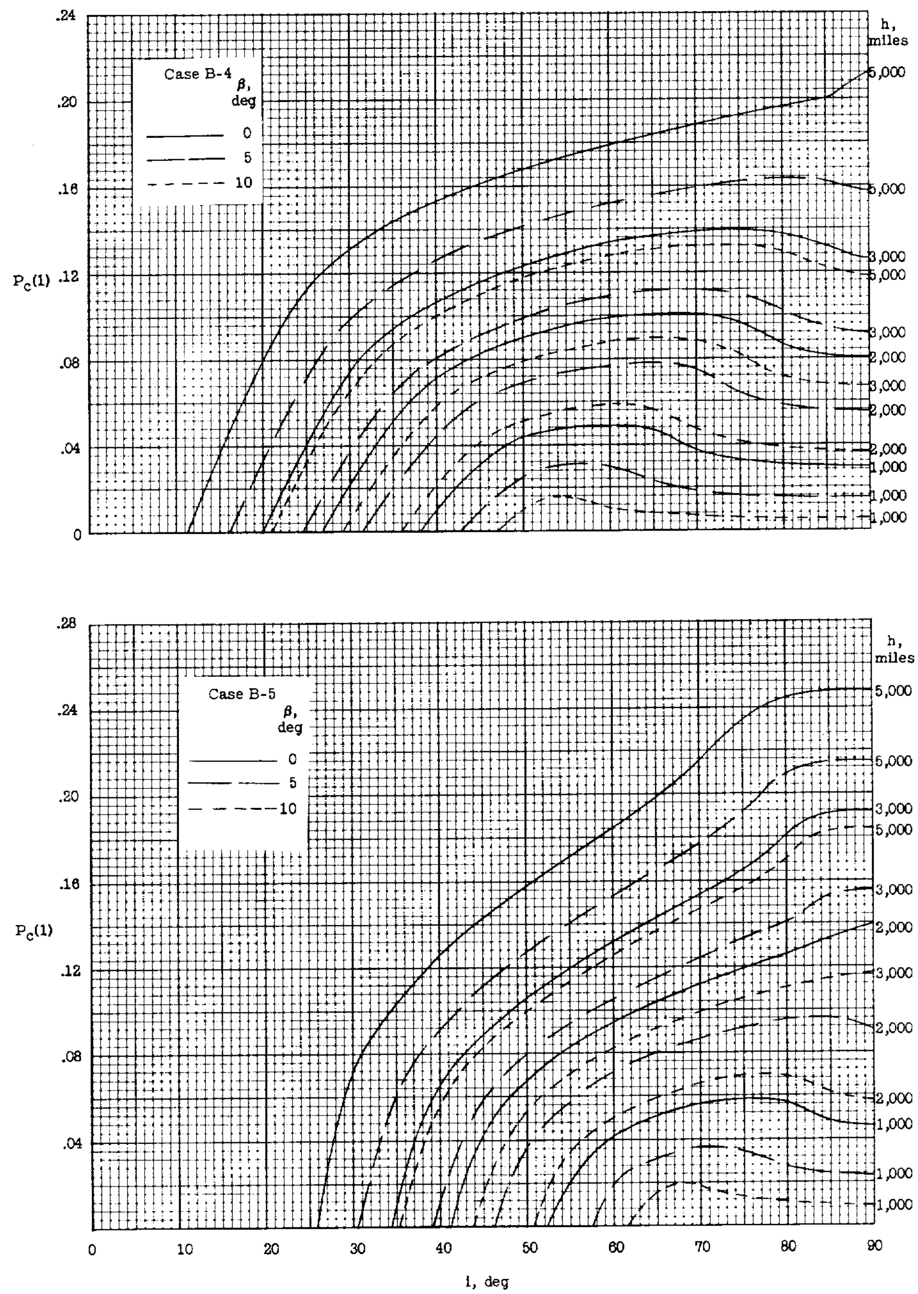

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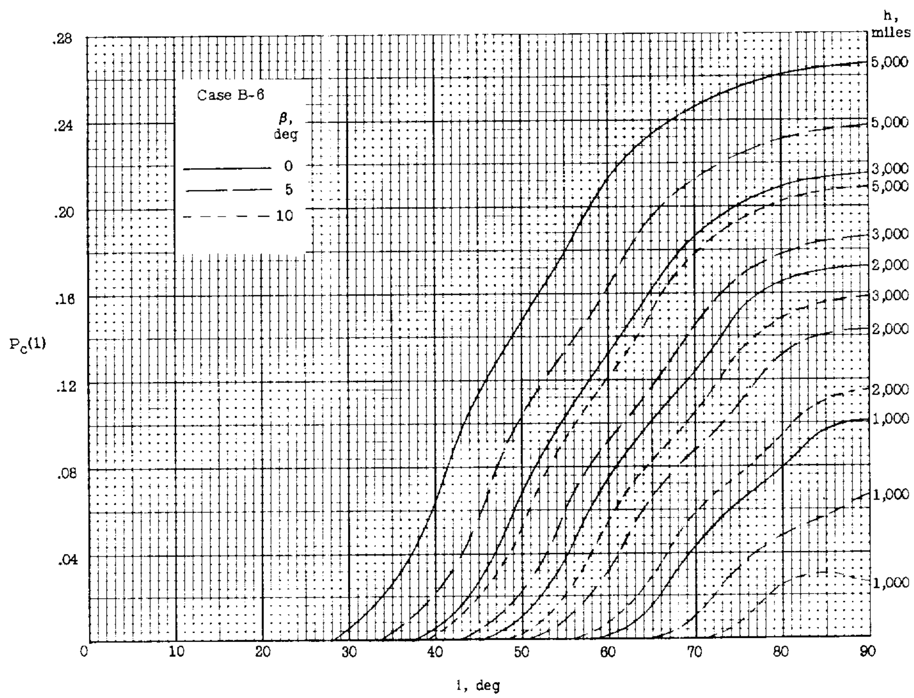

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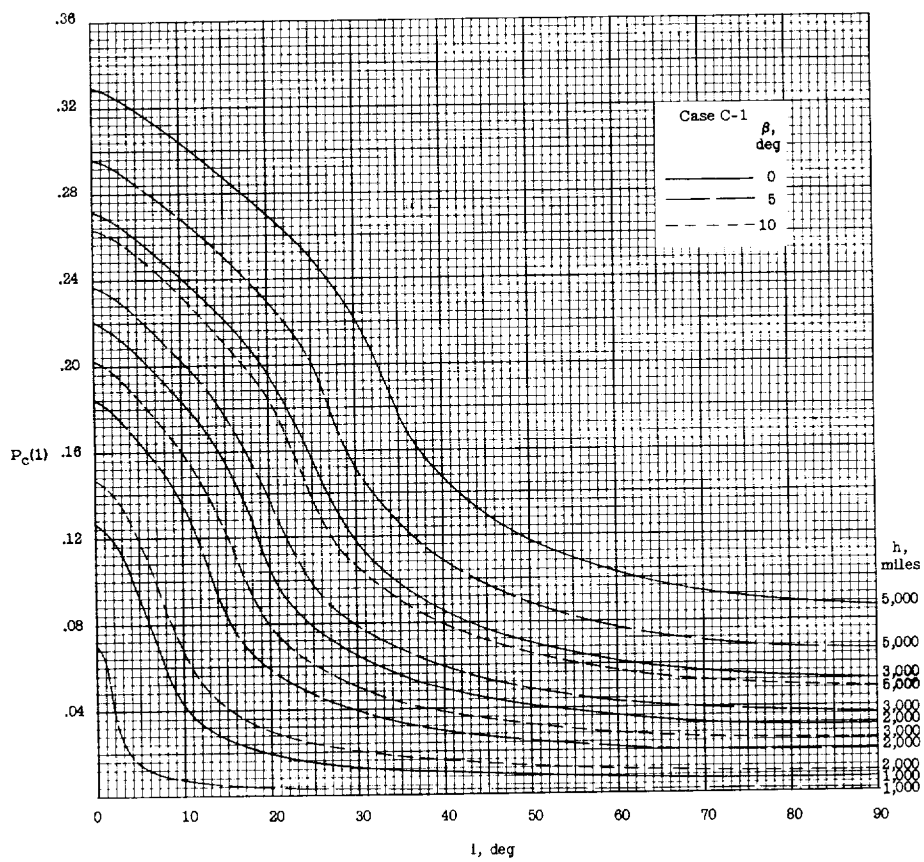

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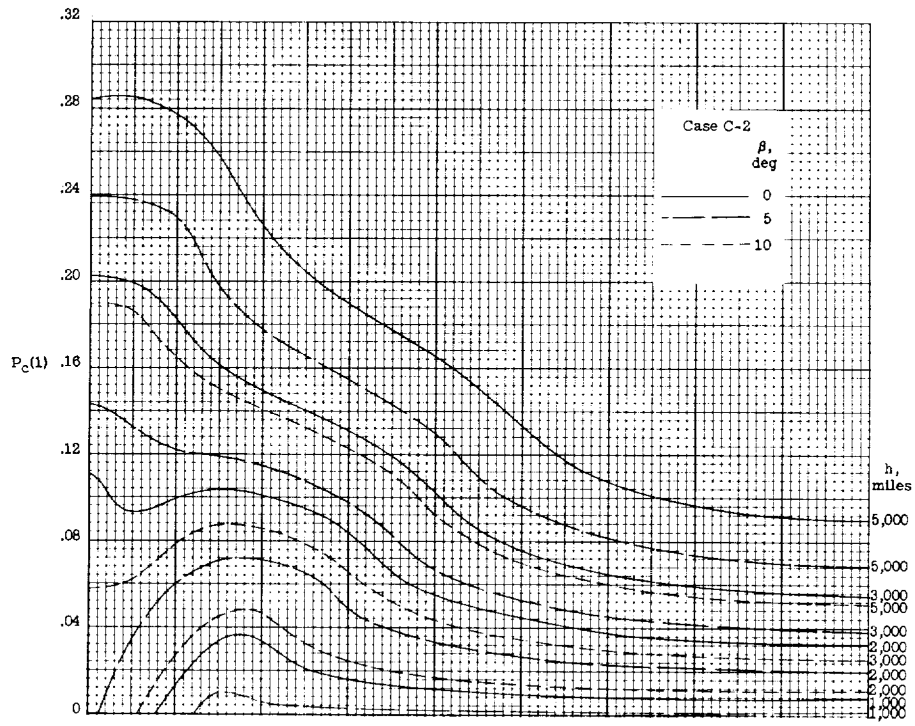

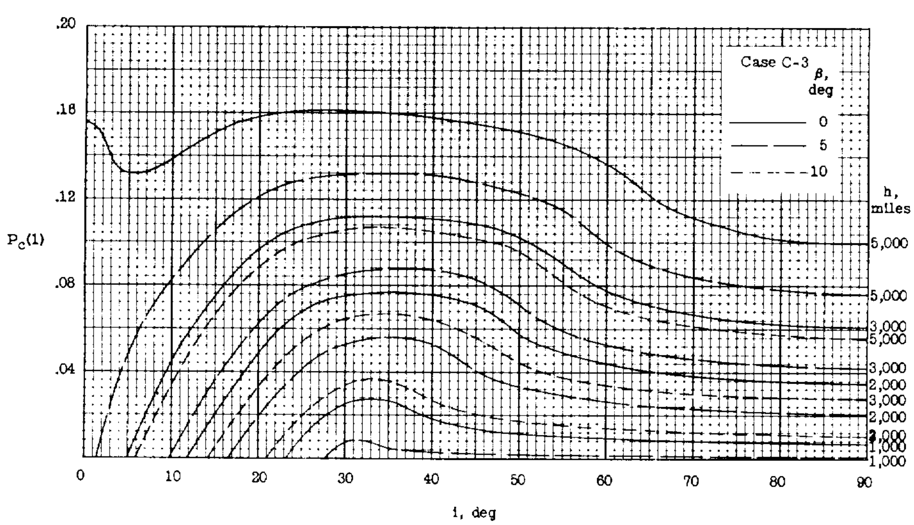

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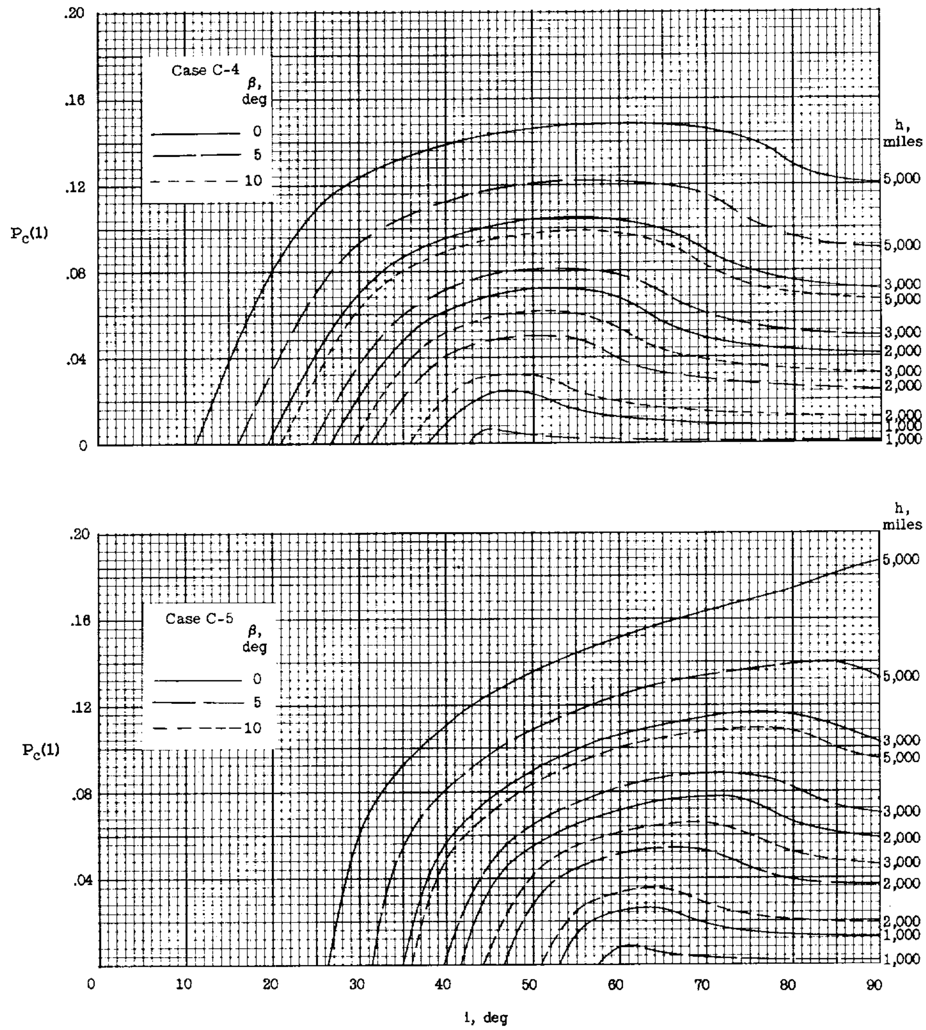

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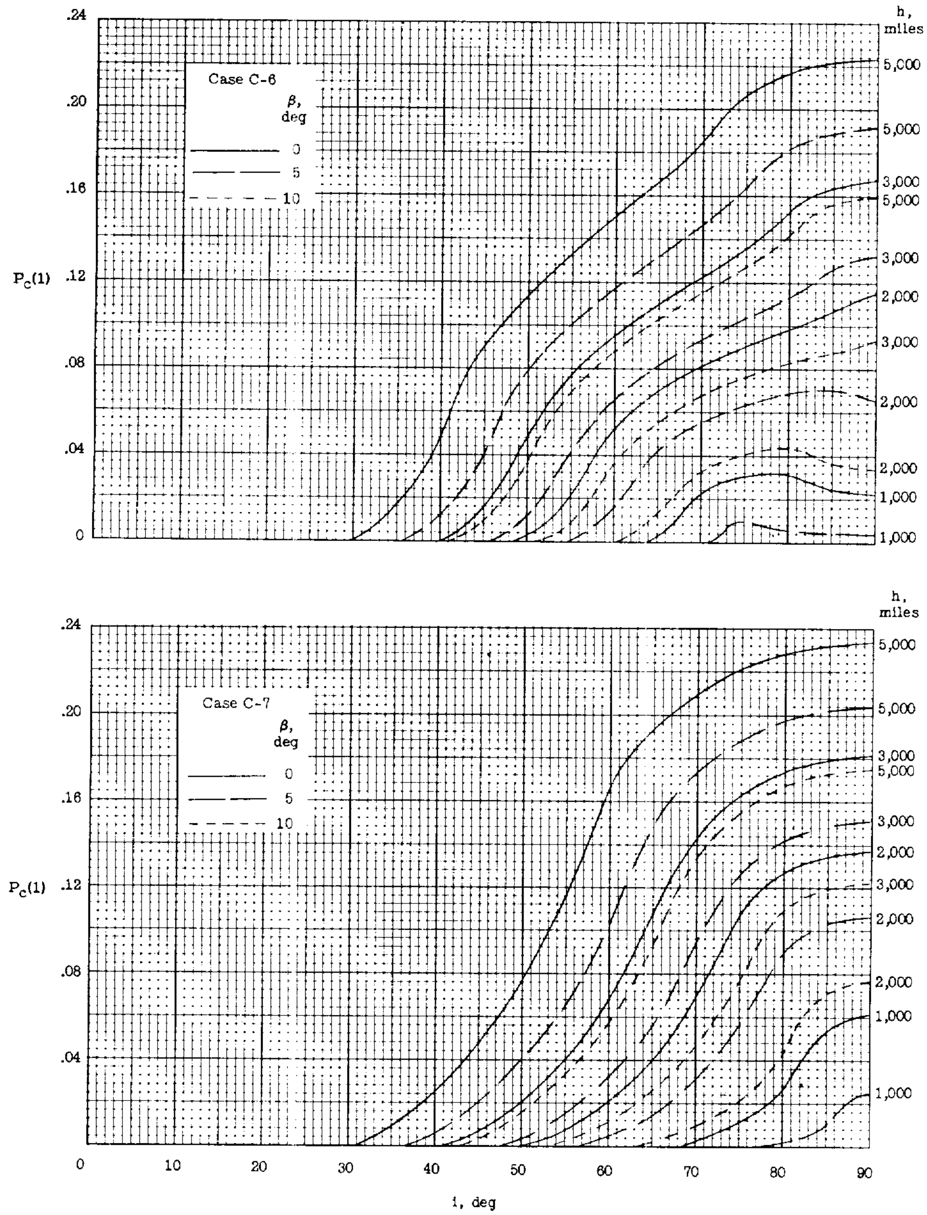

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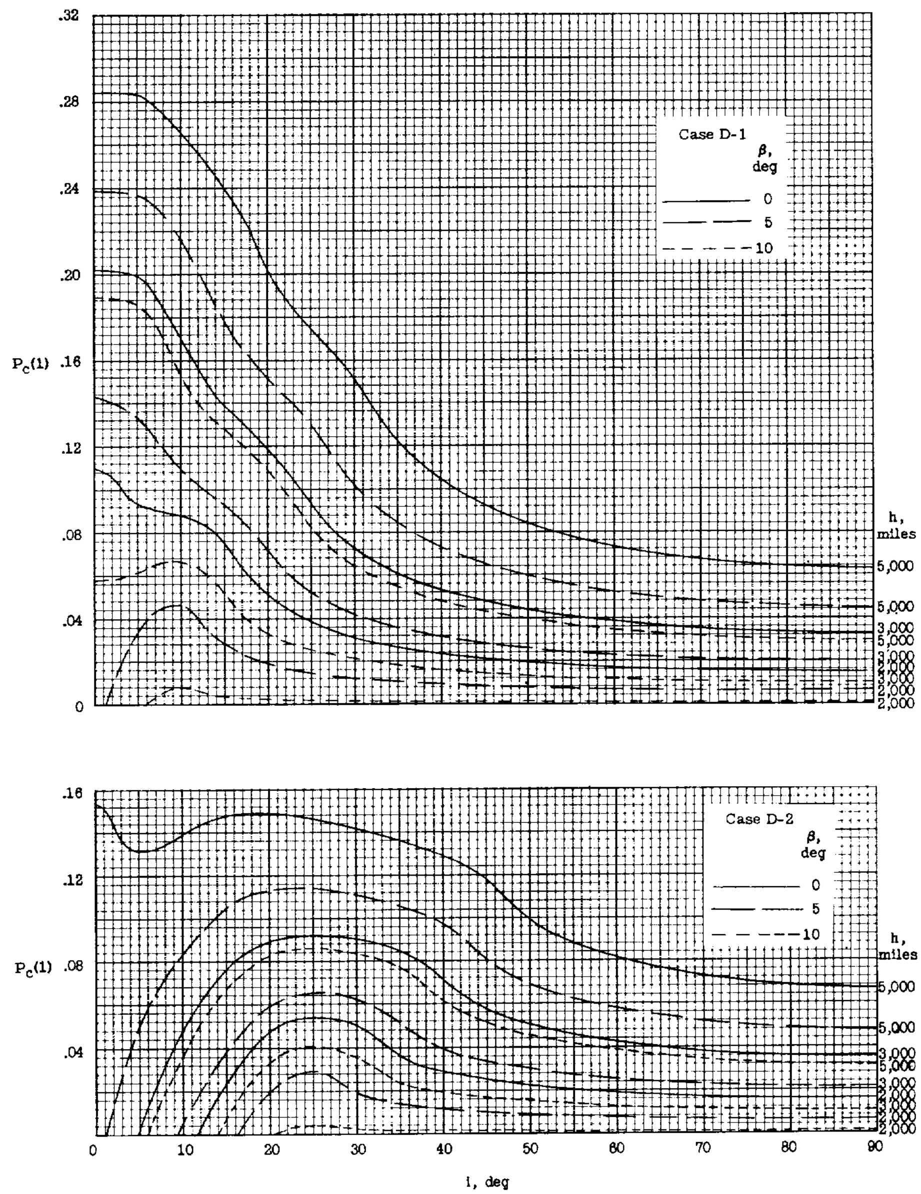

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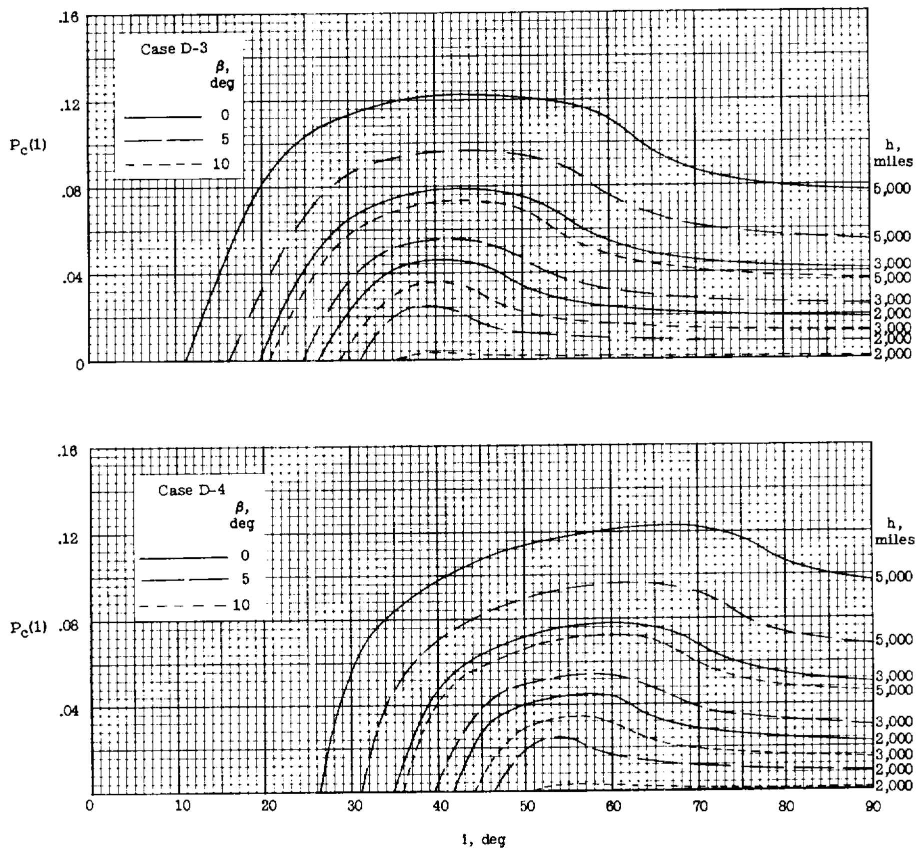

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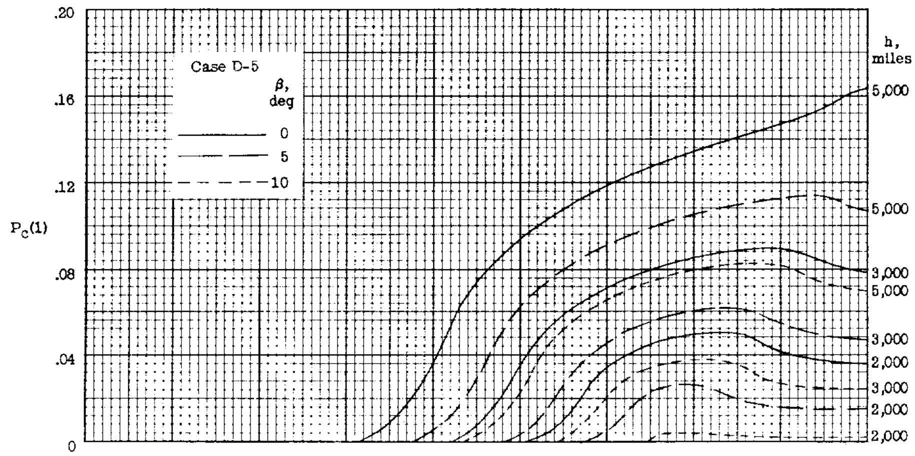

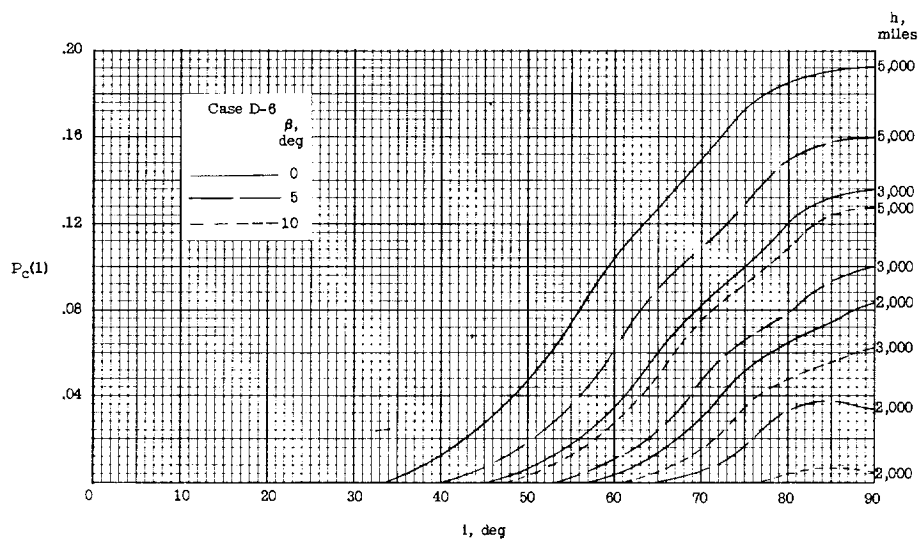

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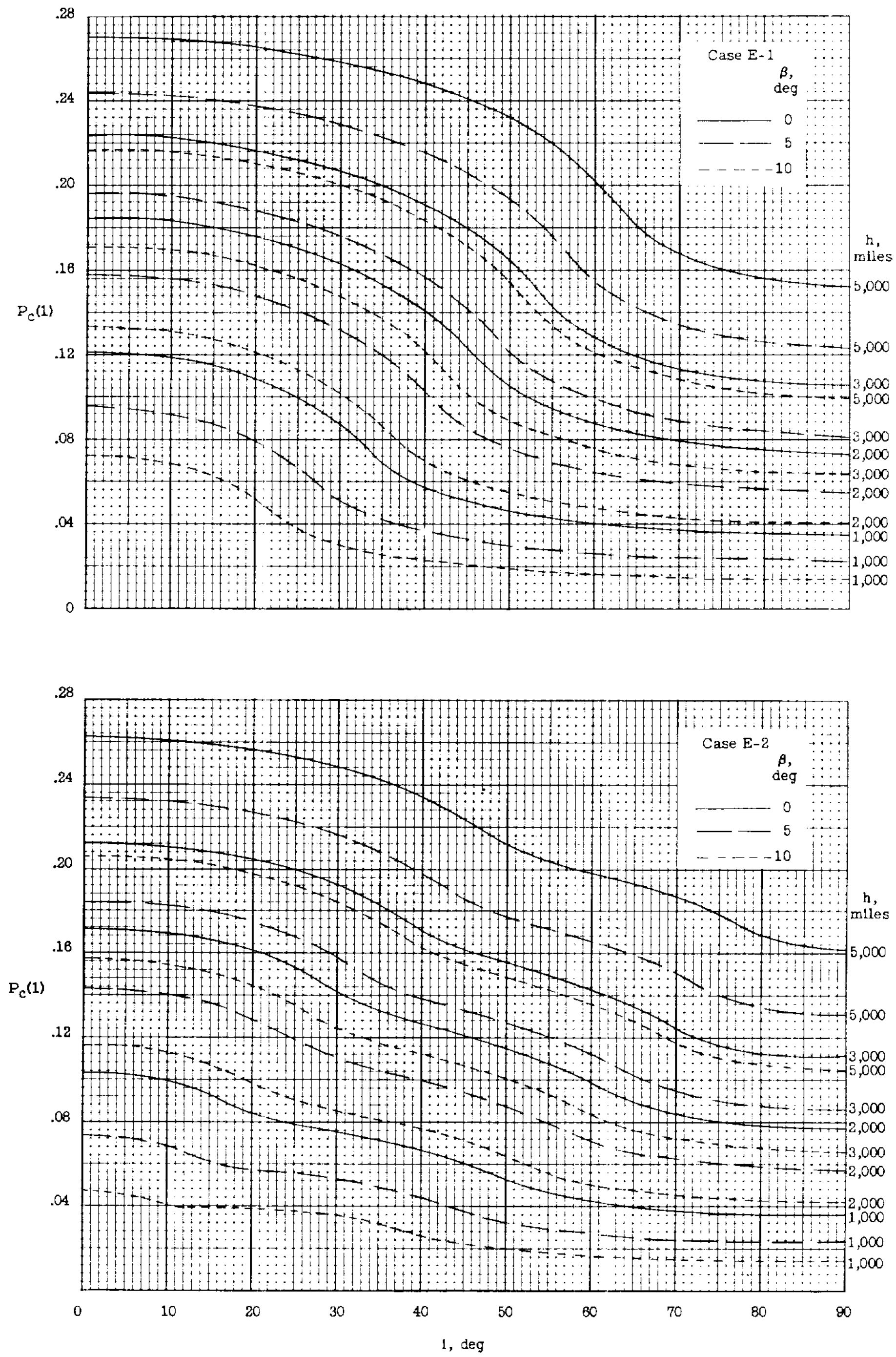

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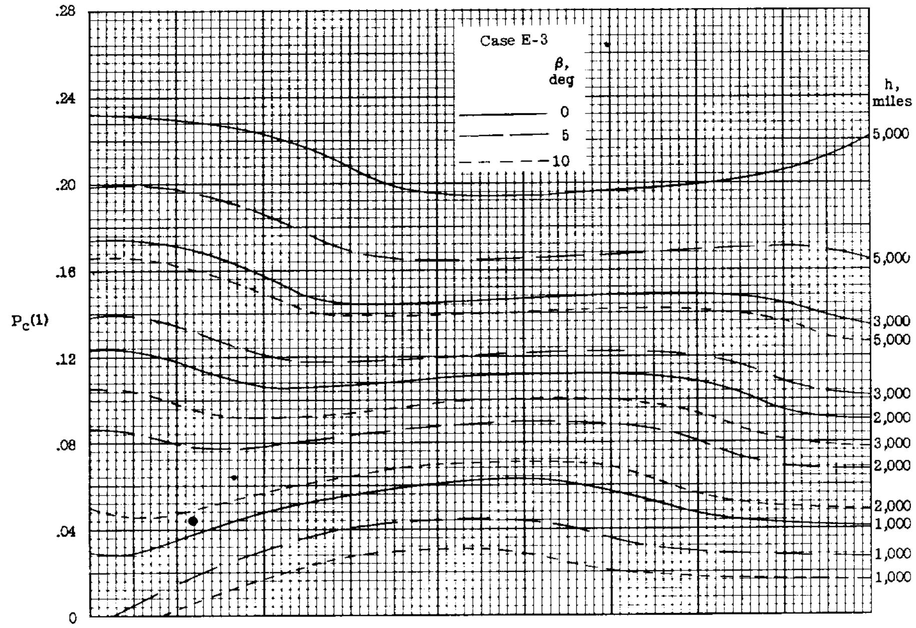

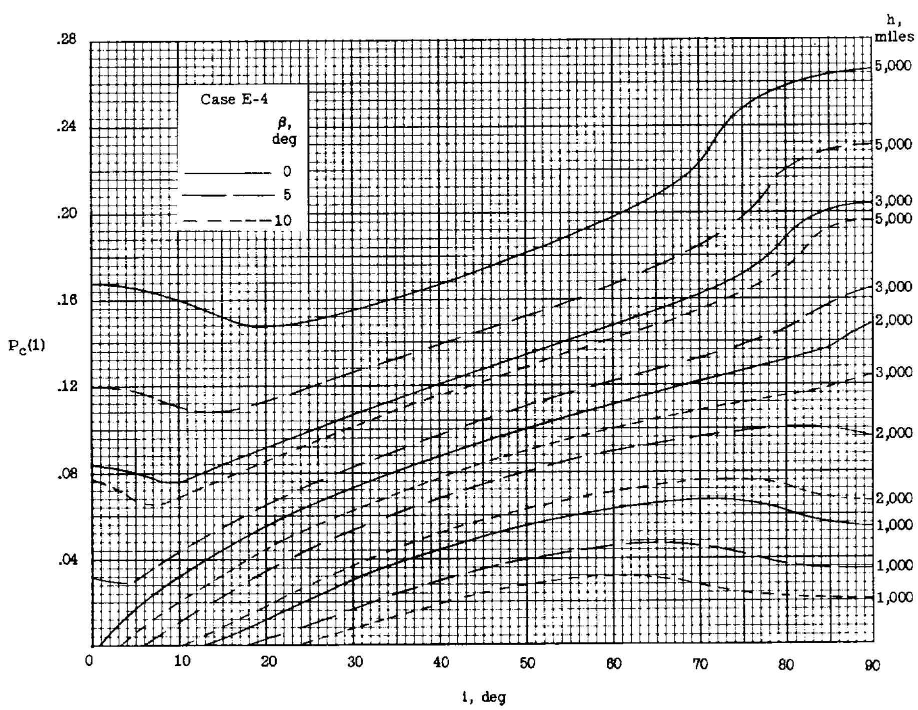

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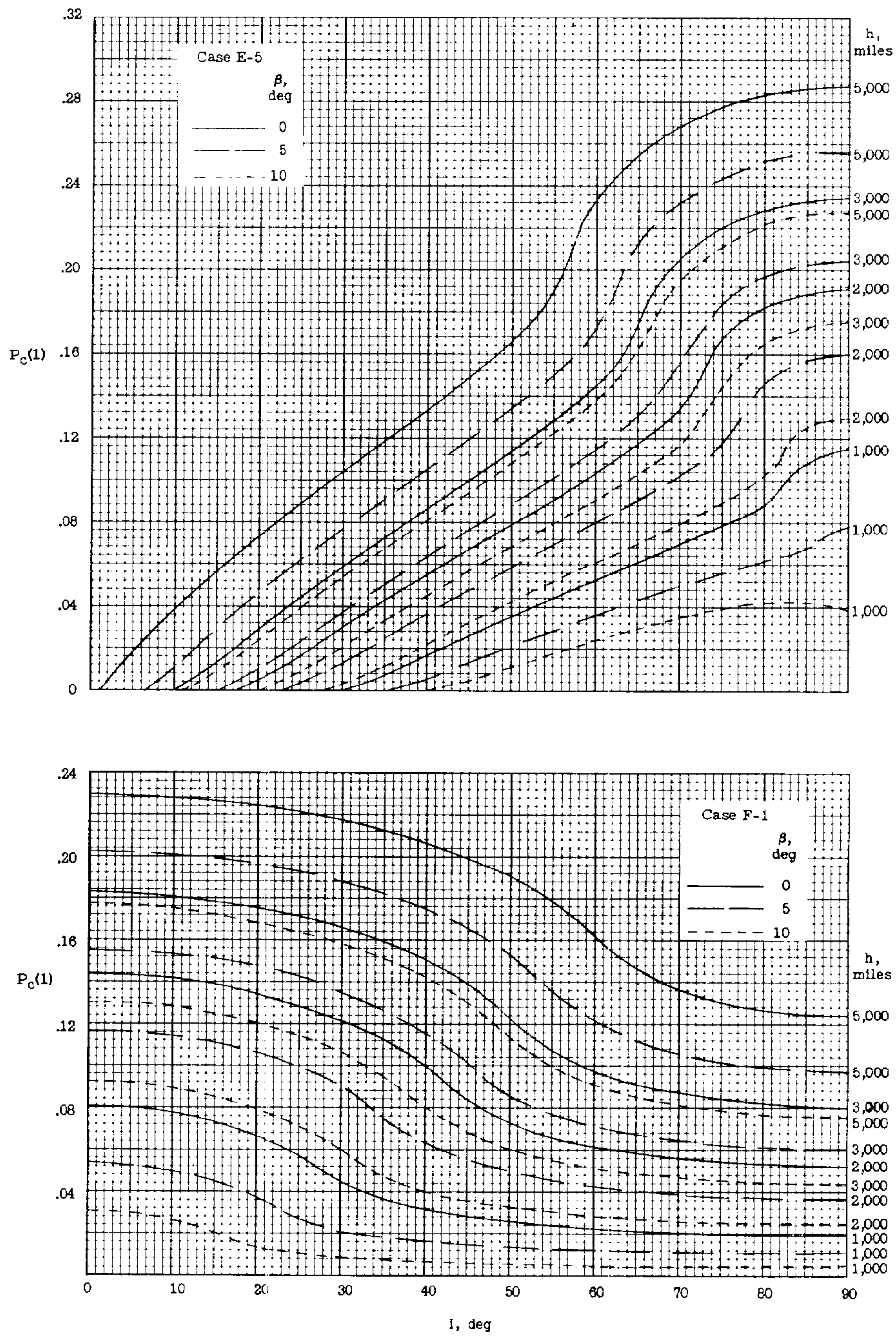

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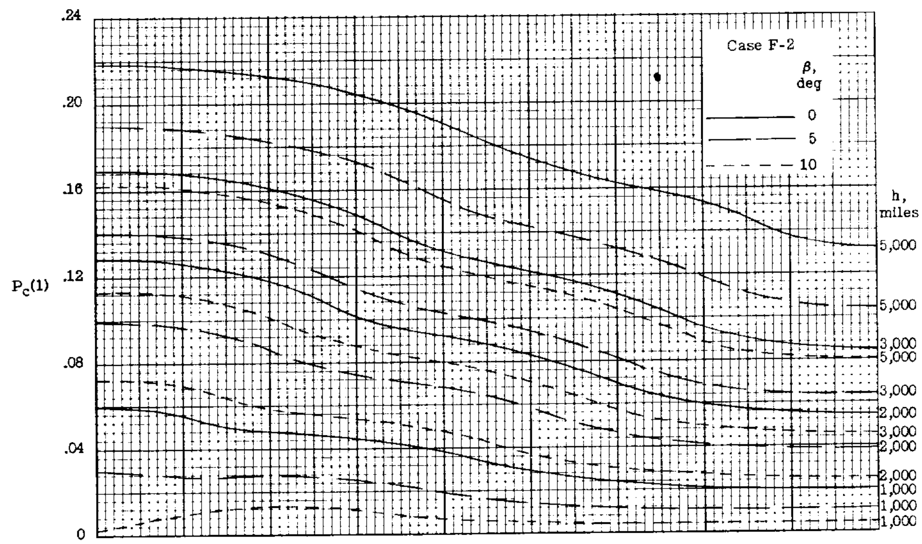

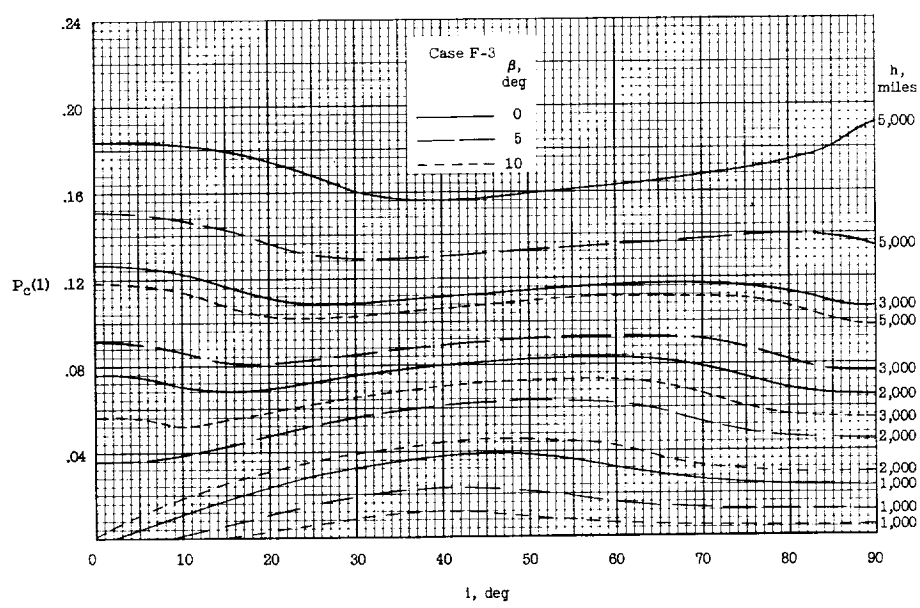

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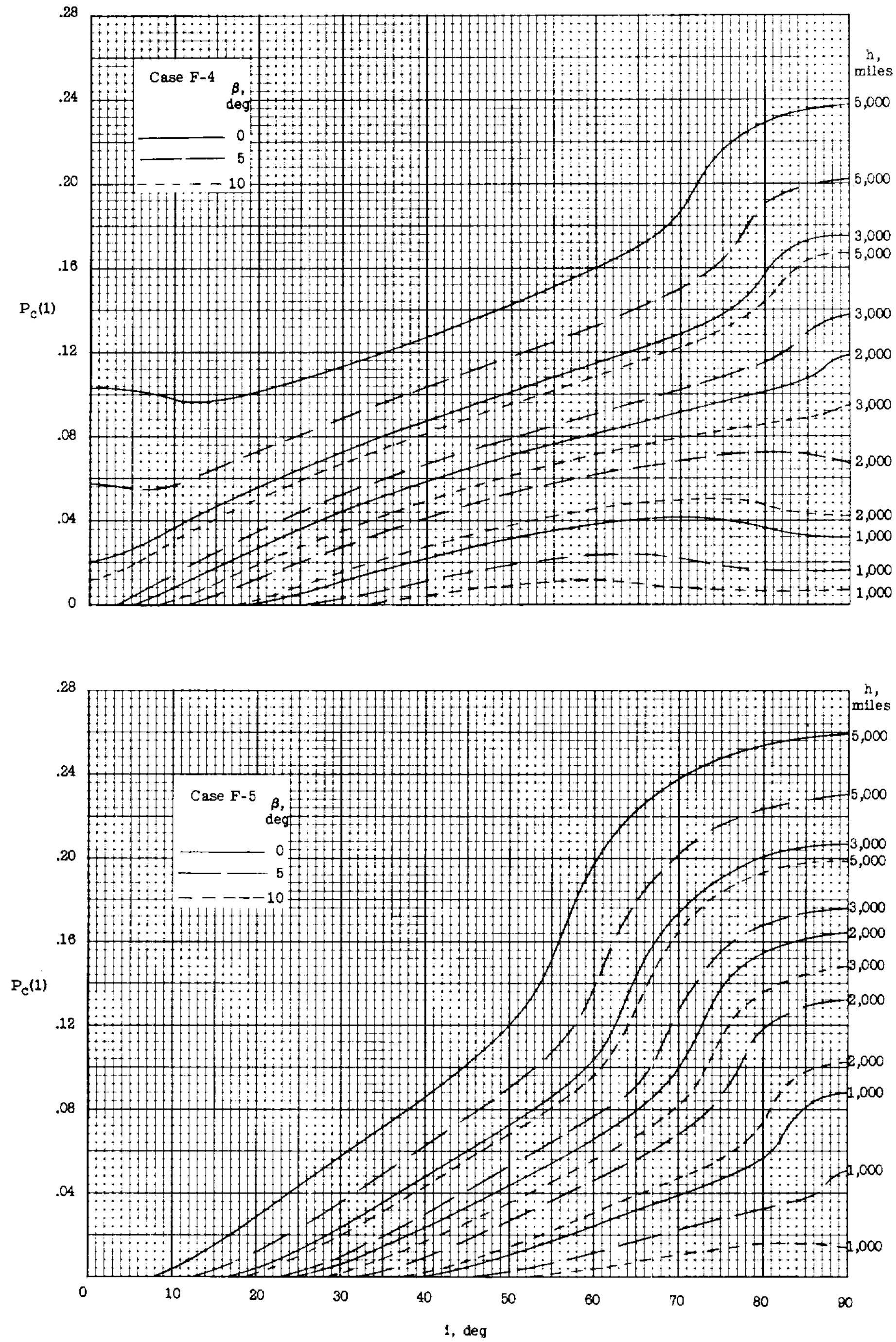

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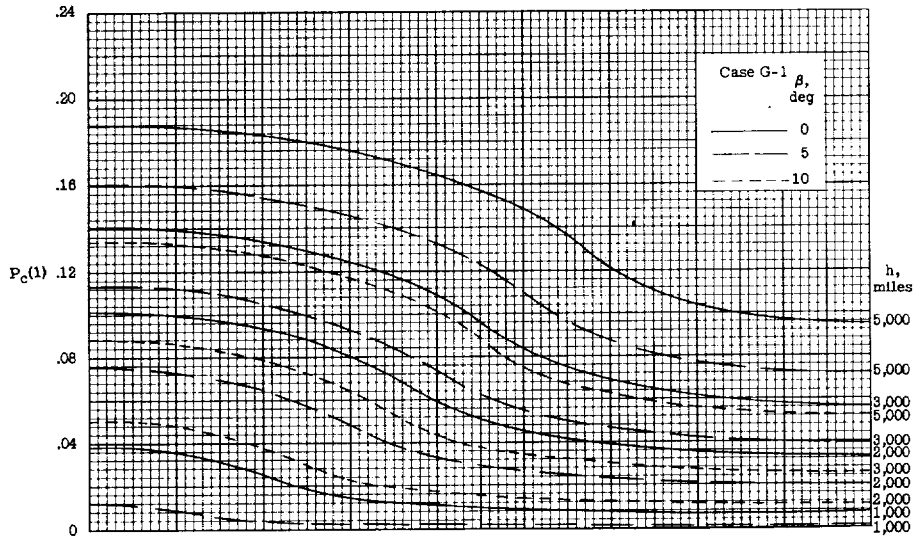

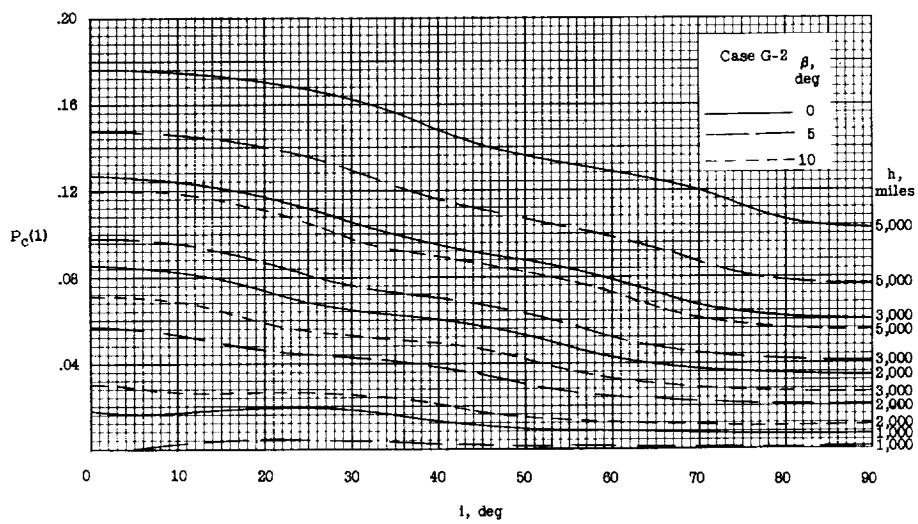

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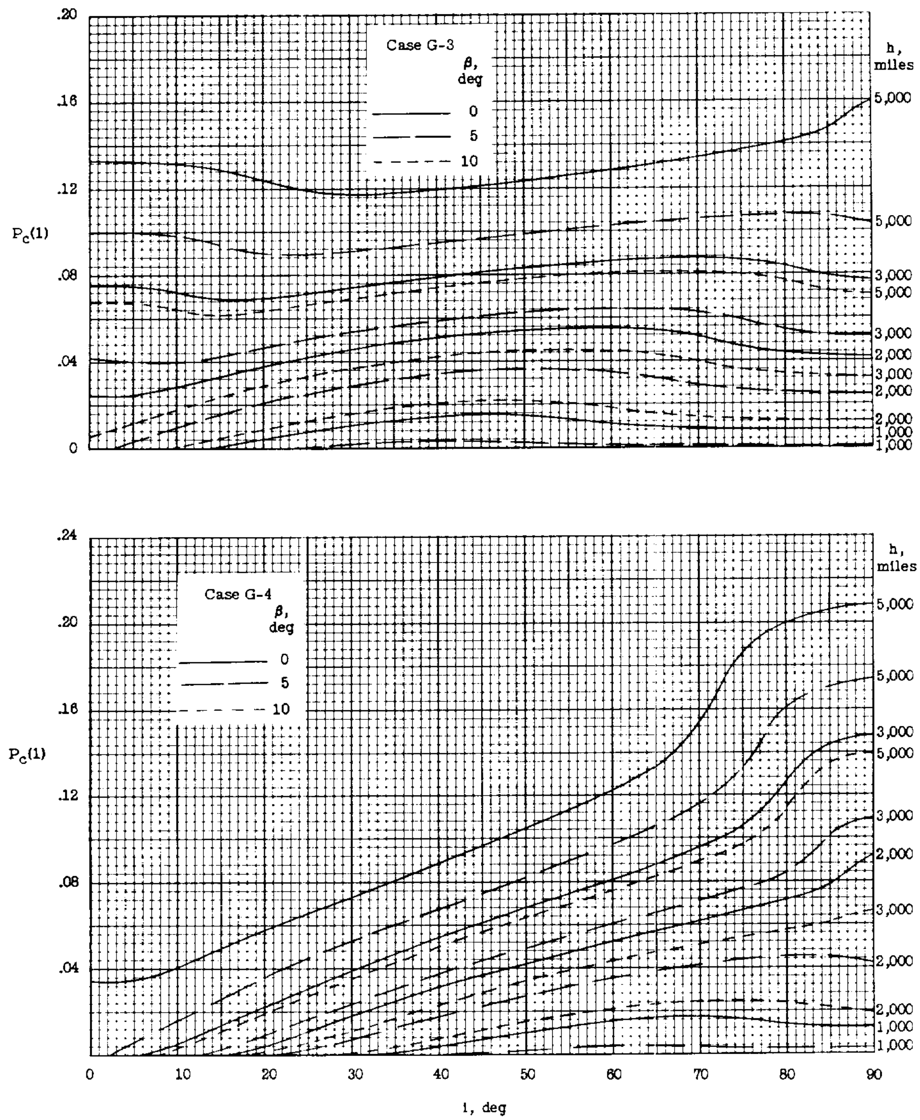

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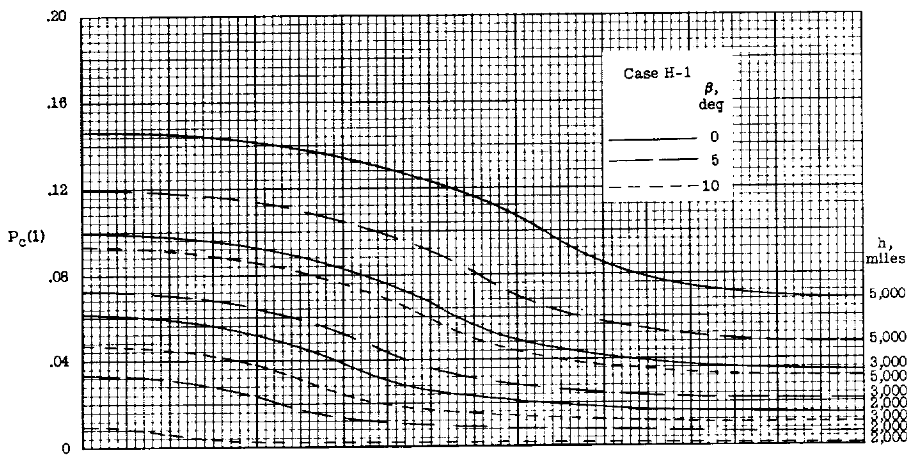

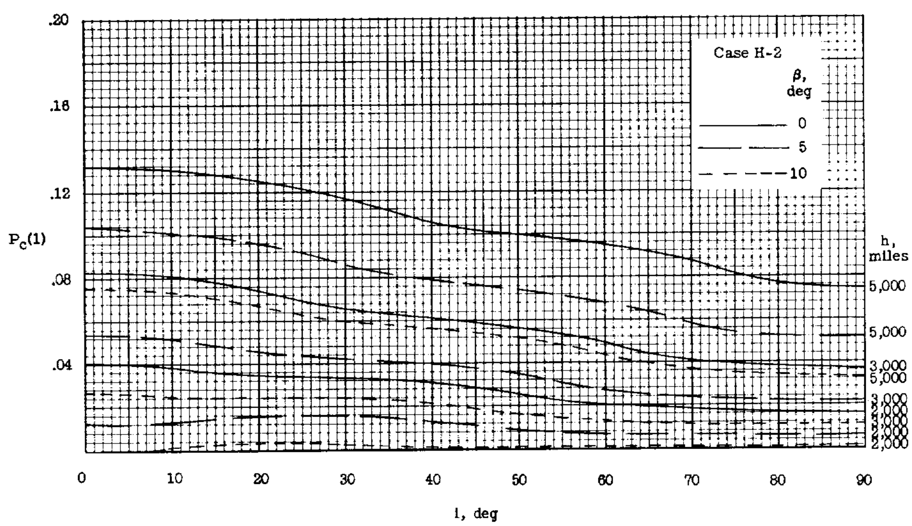

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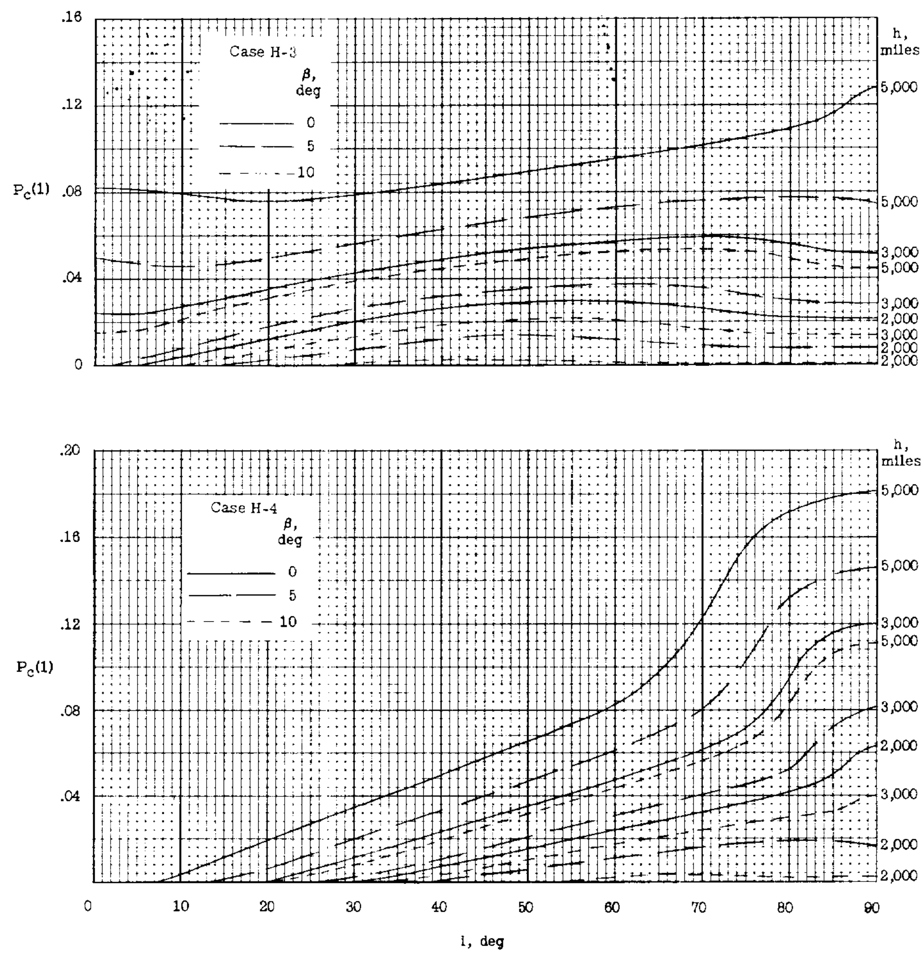

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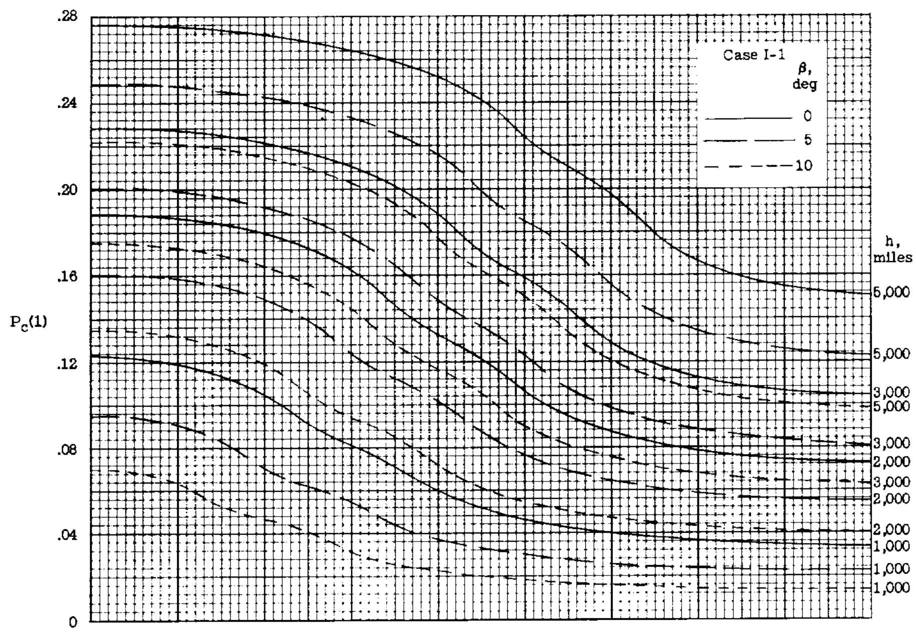

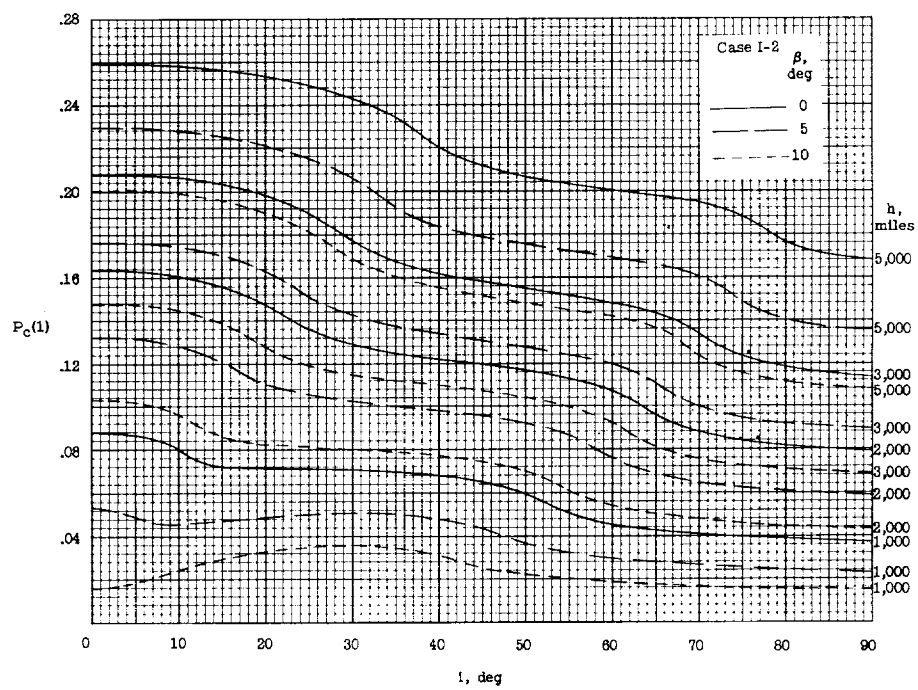

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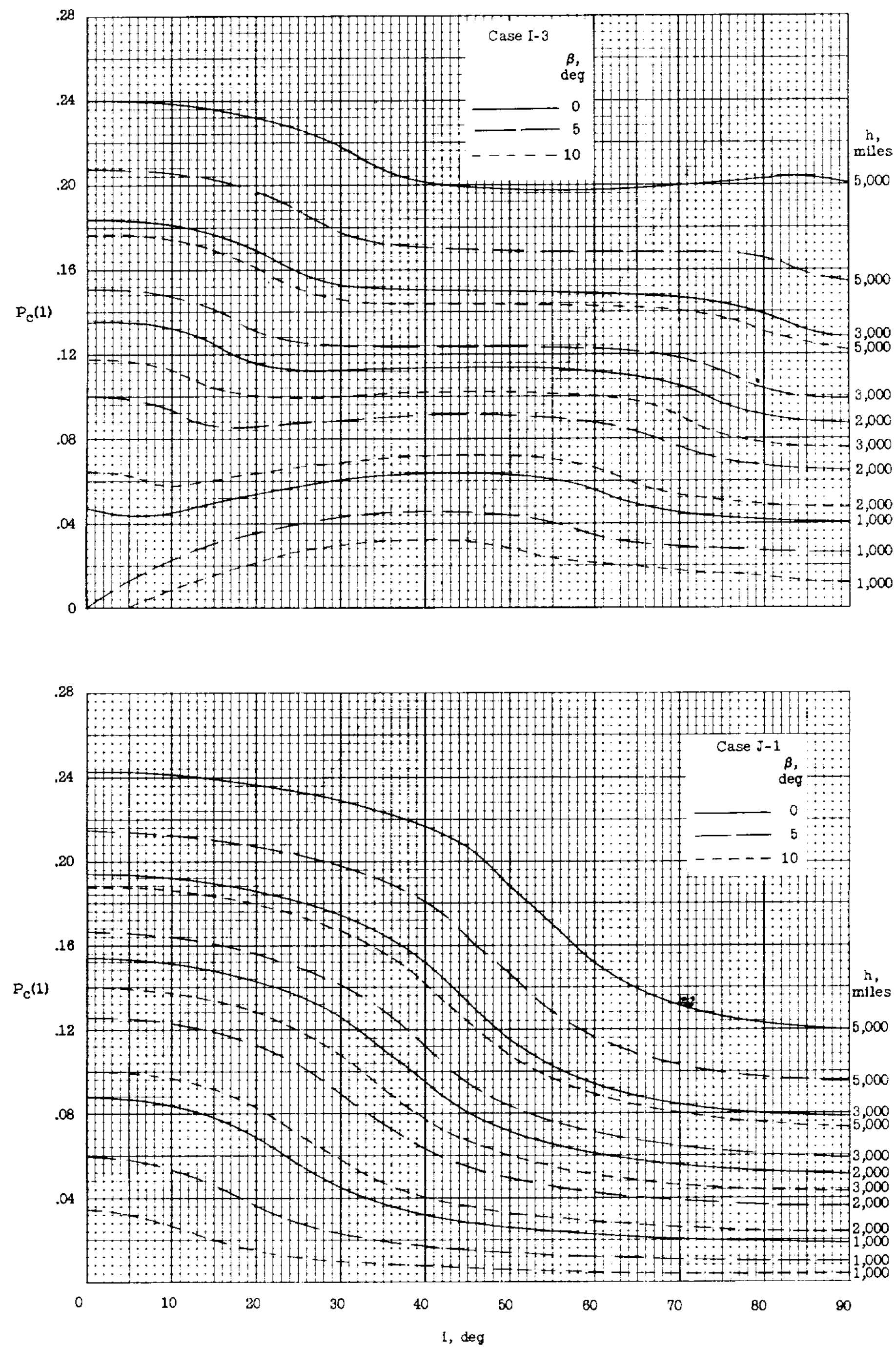

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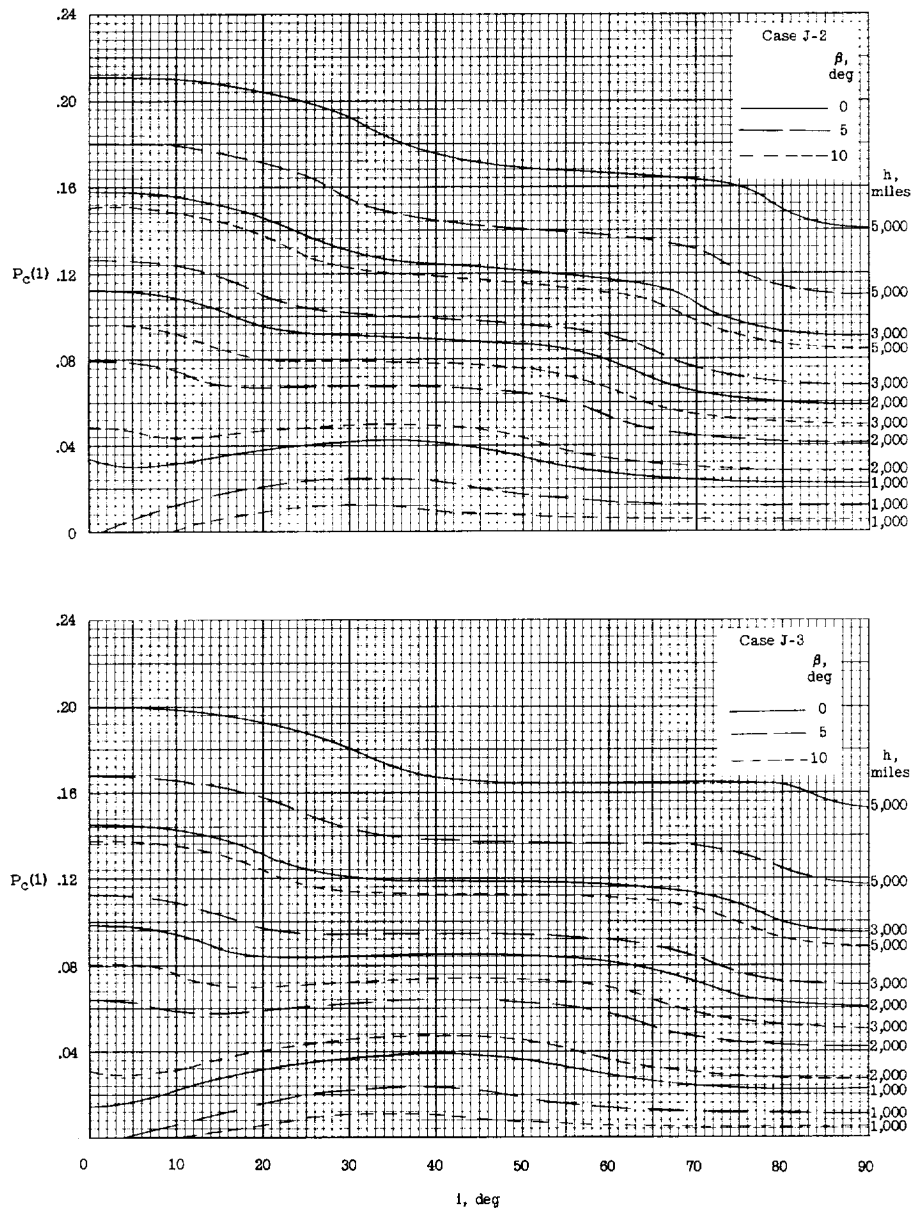

Figure 5.- Continued.

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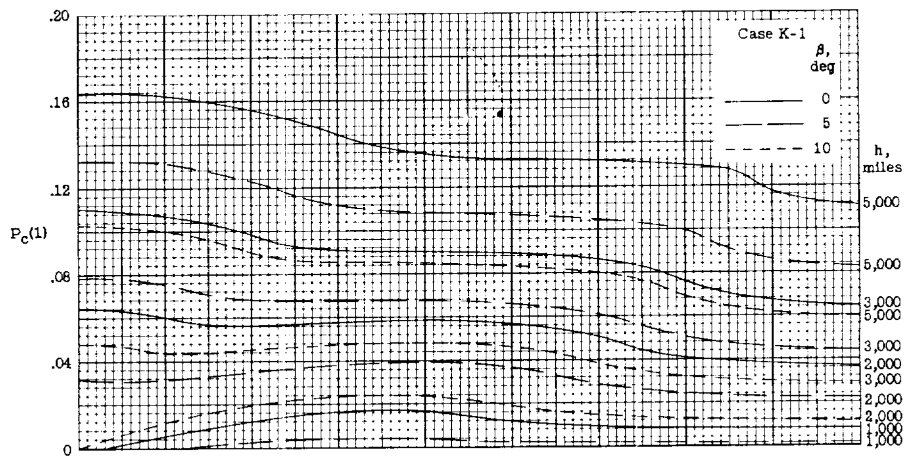

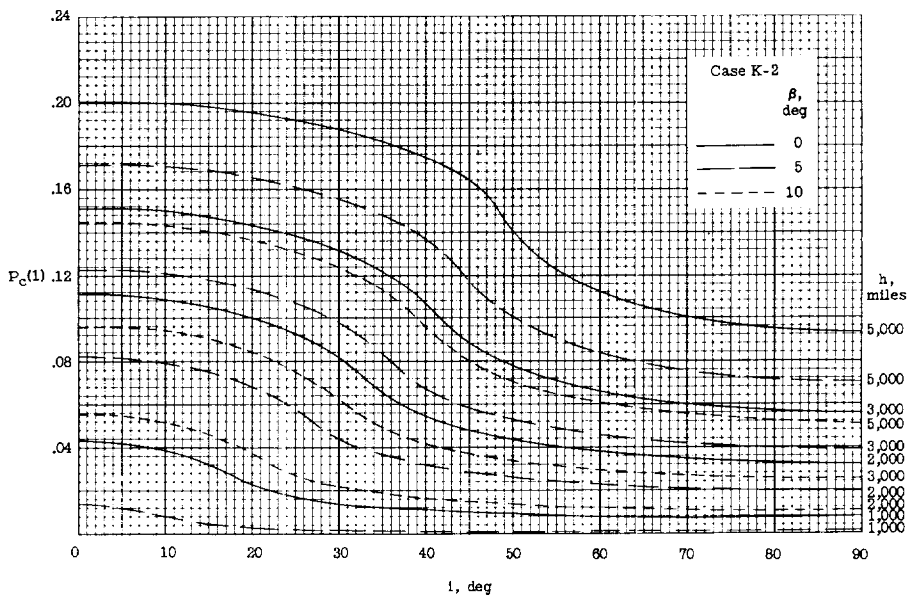

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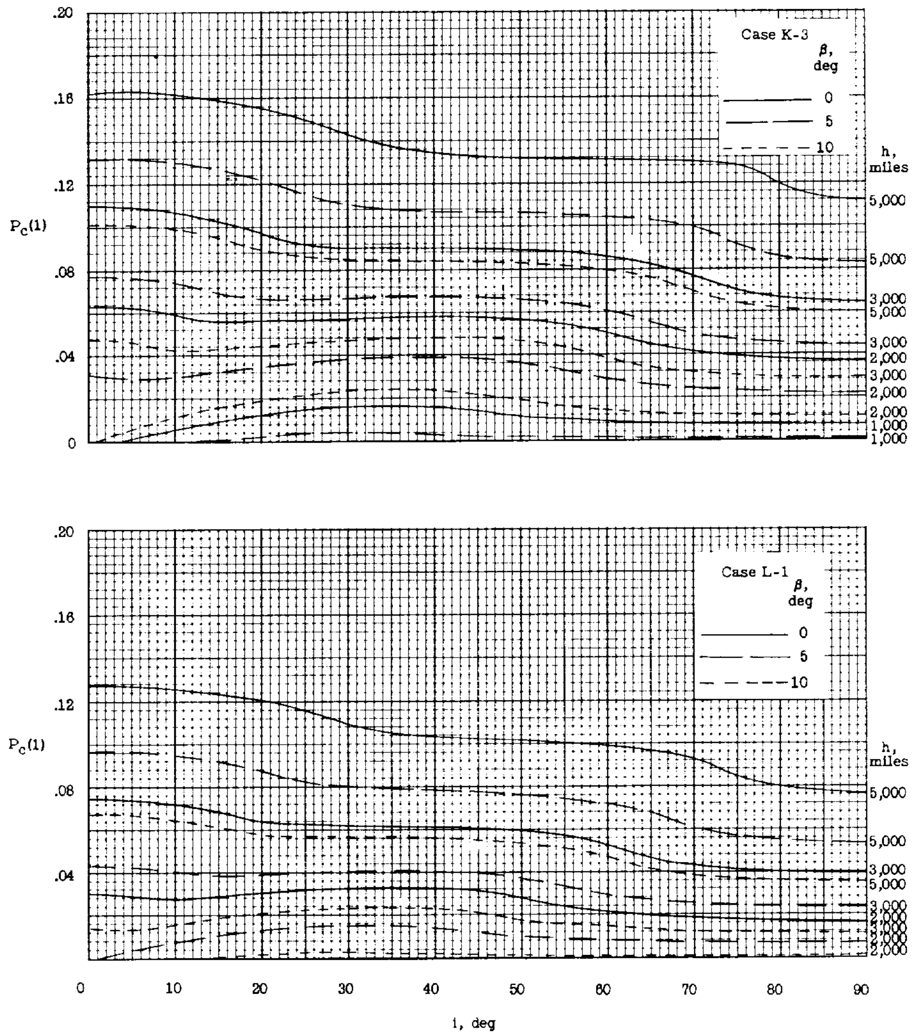

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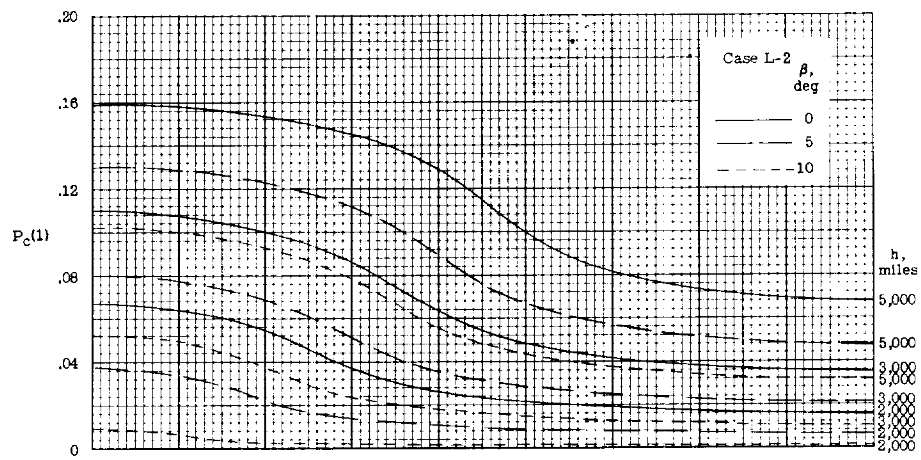

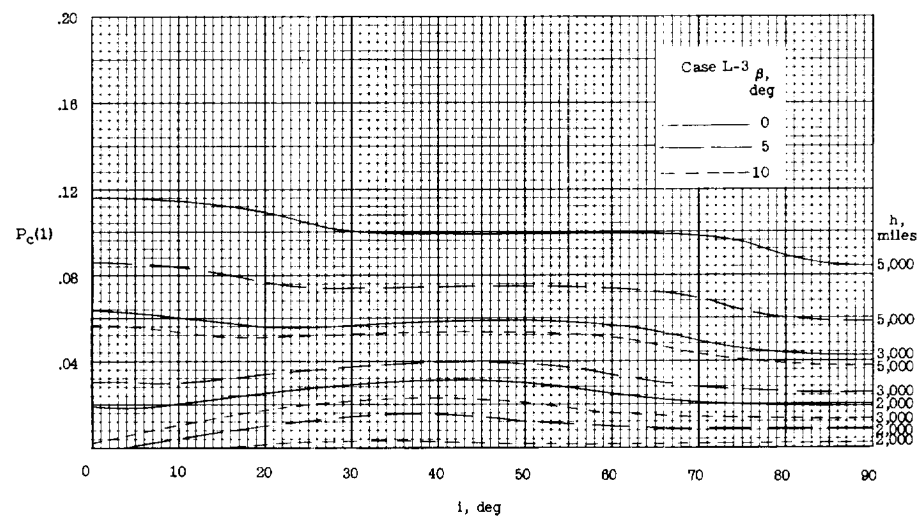

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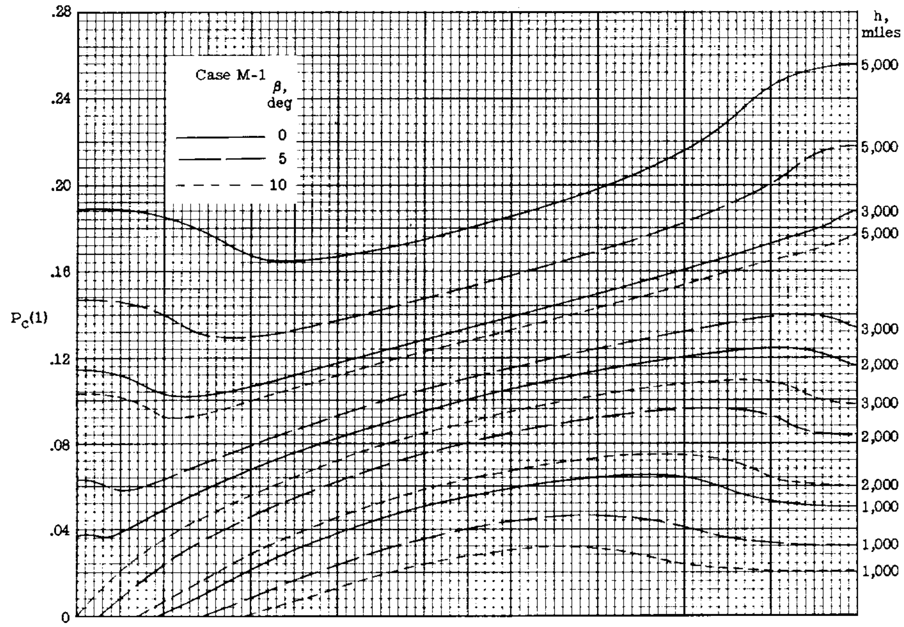

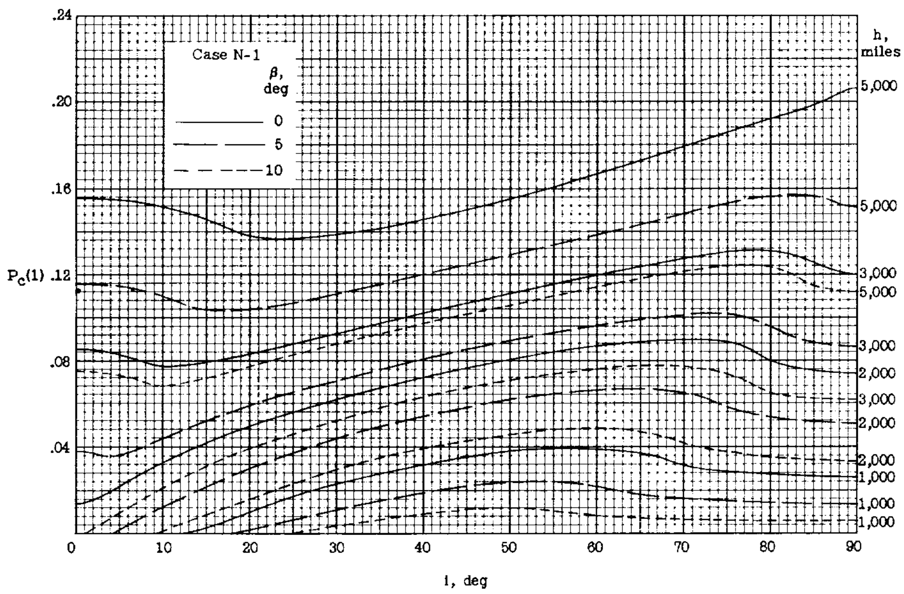

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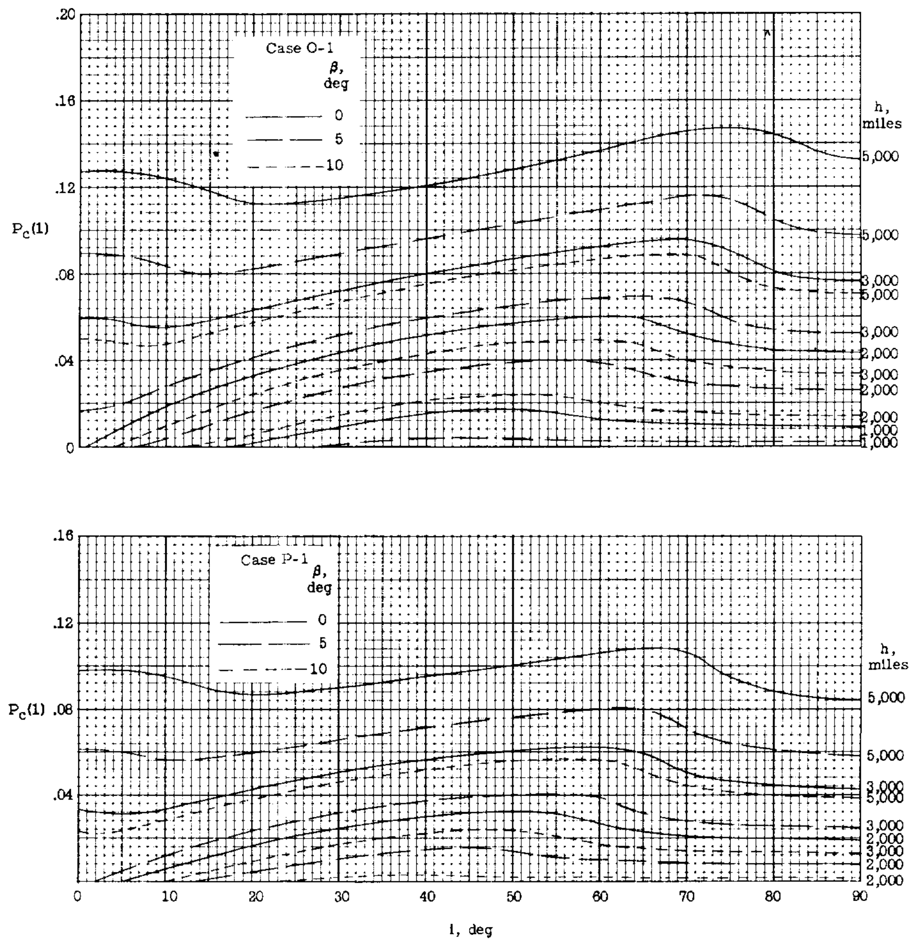

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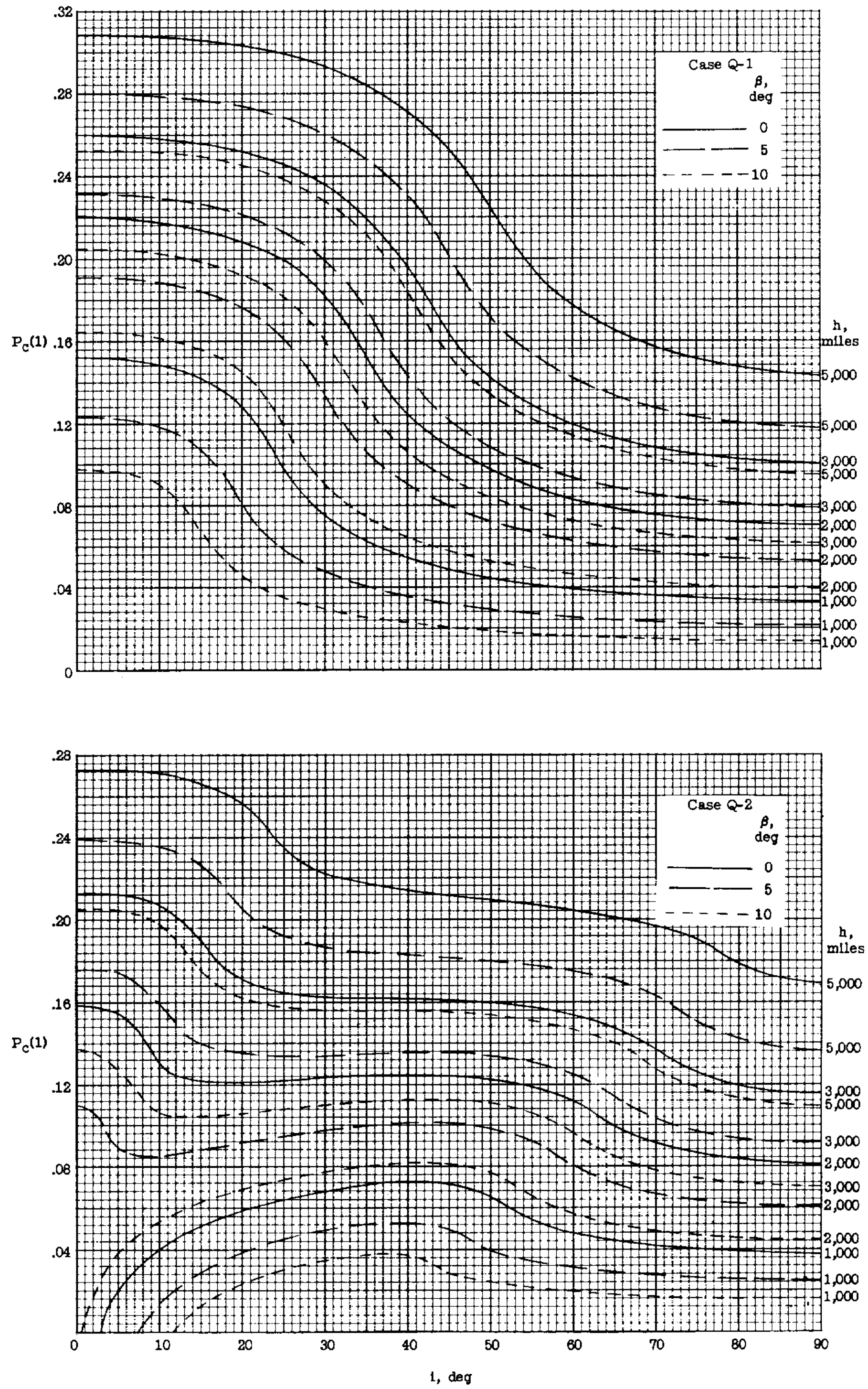

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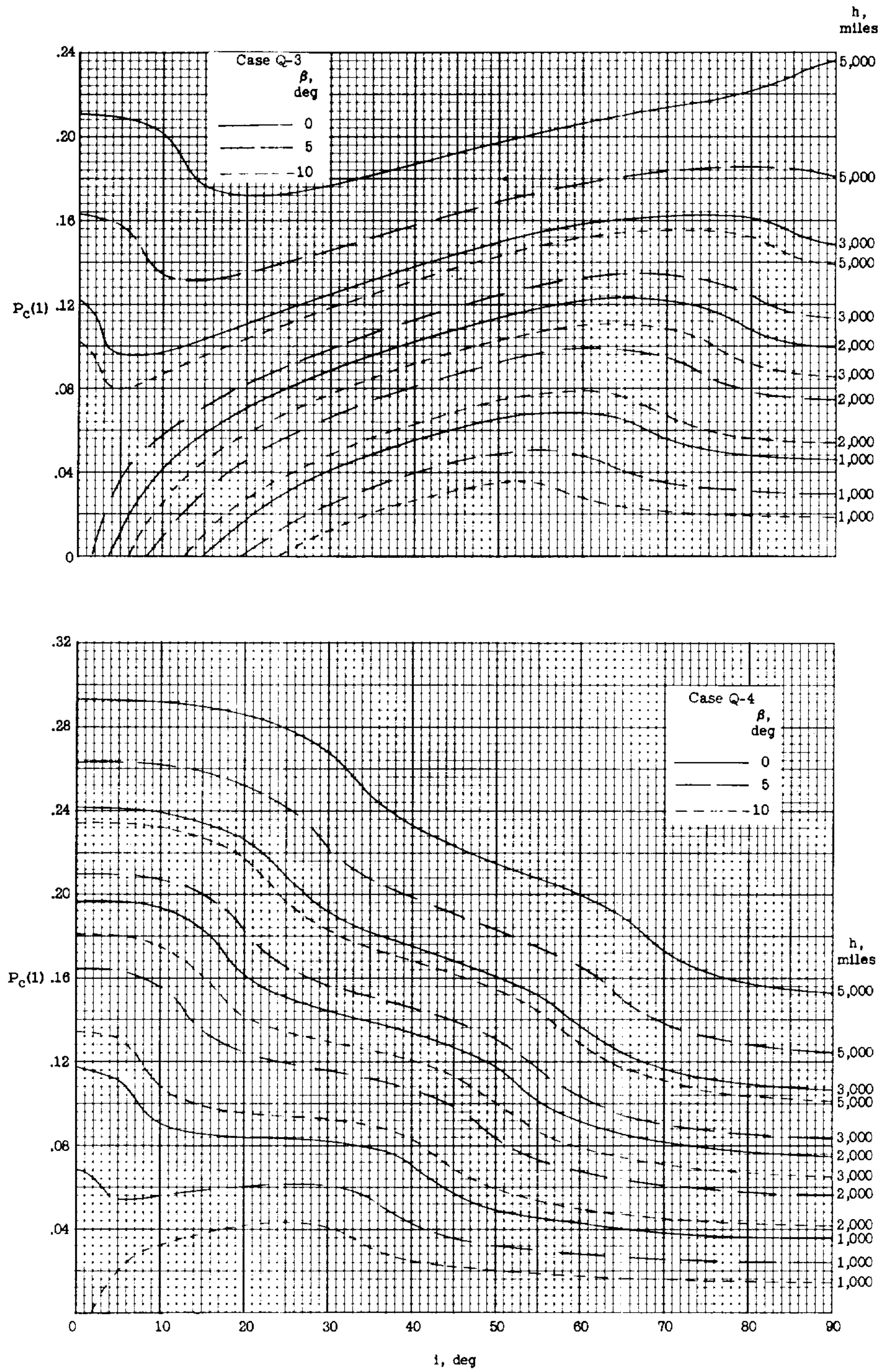

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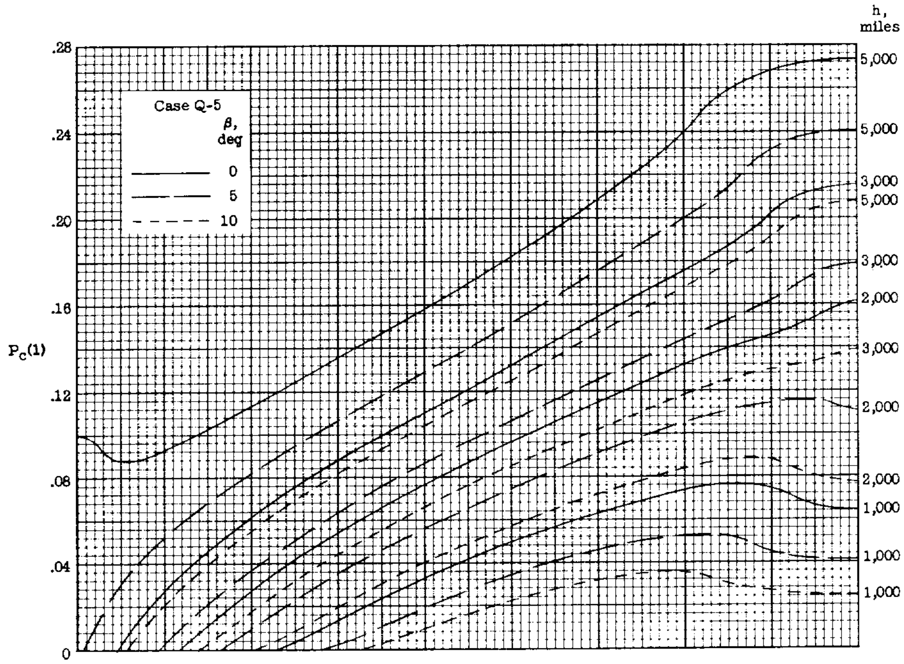

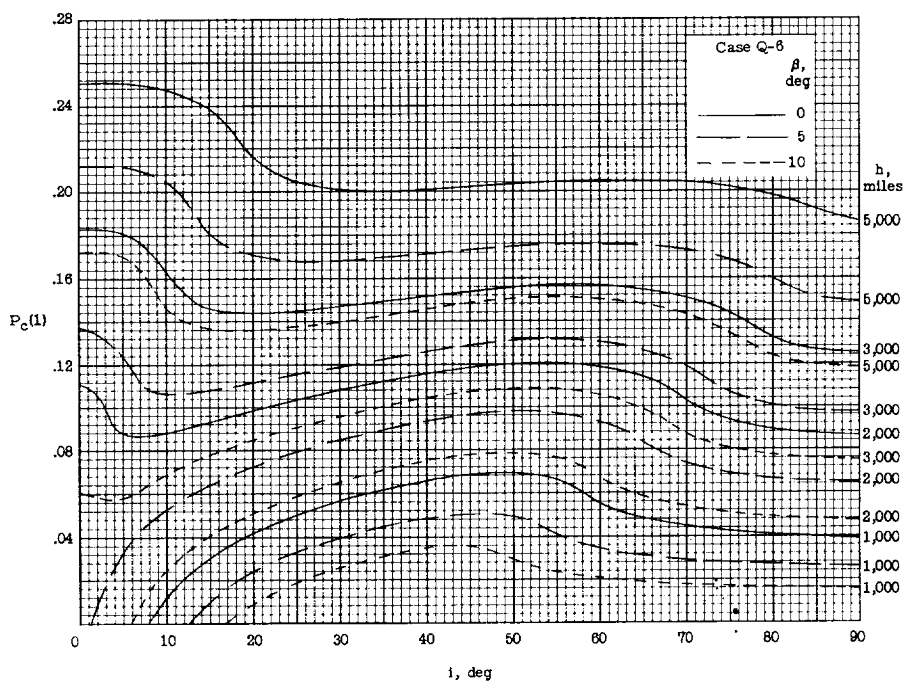

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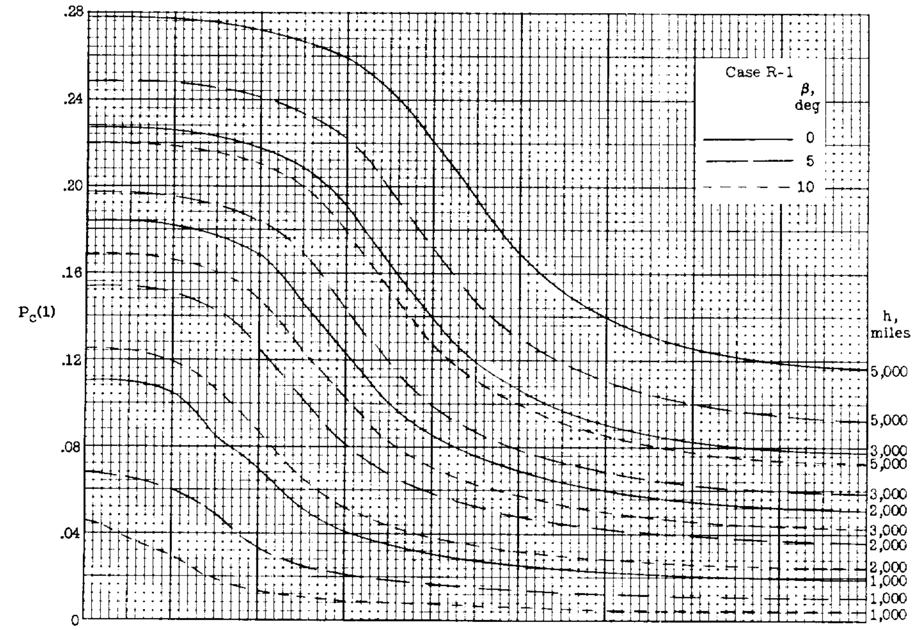

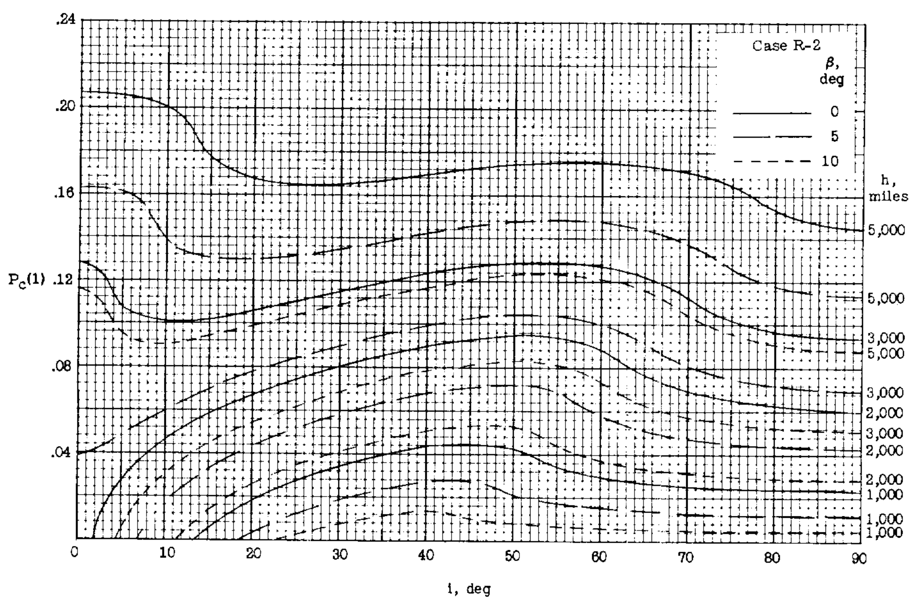

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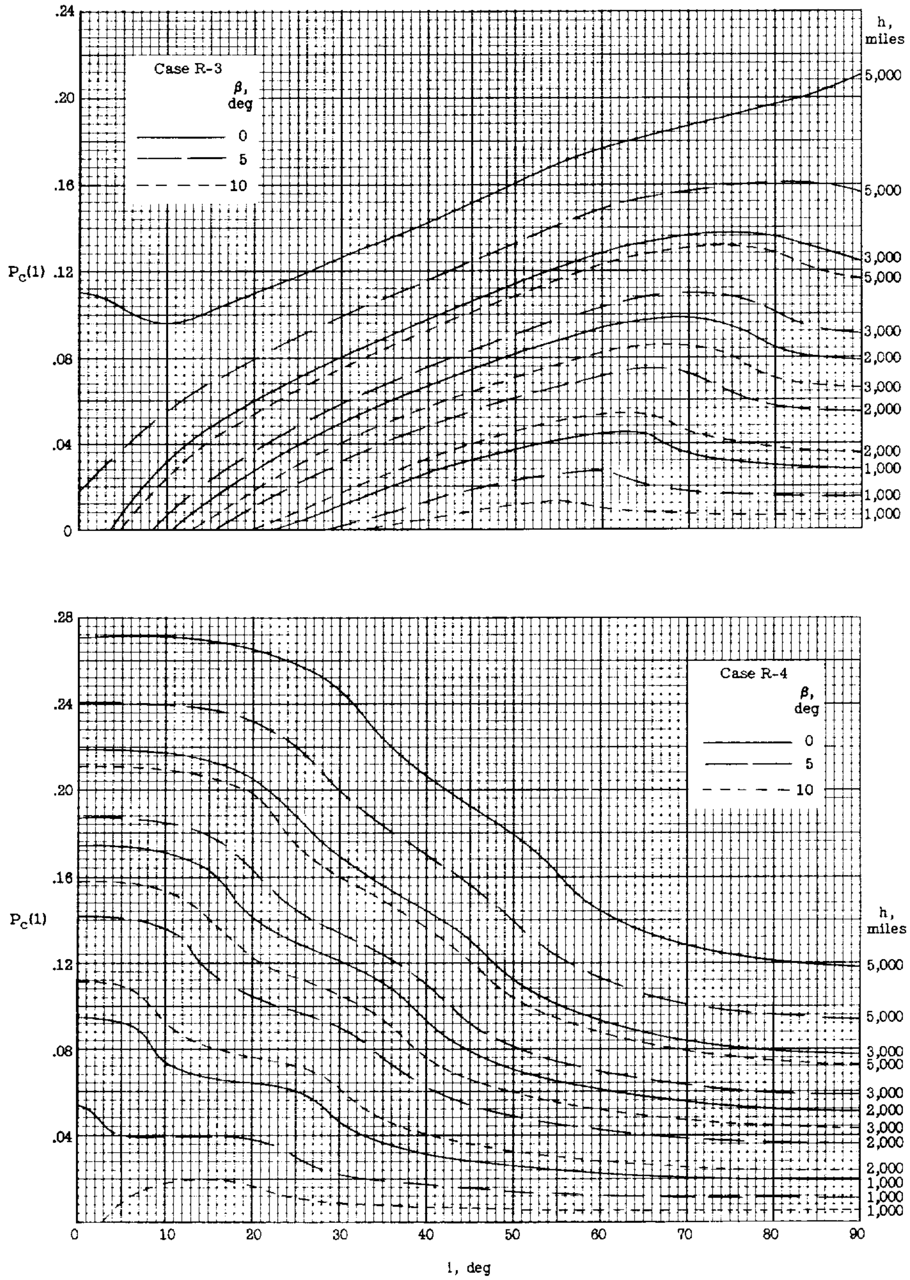

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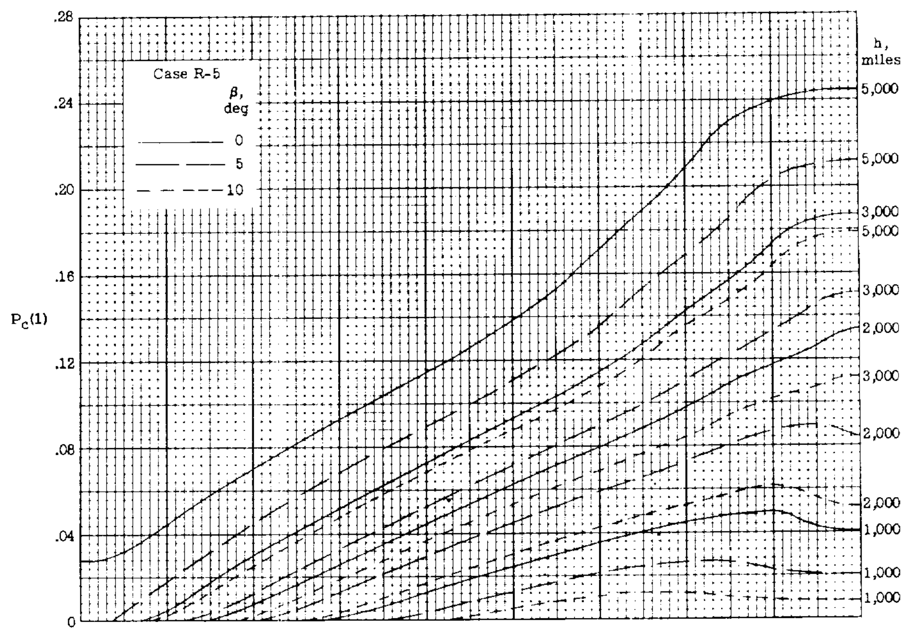

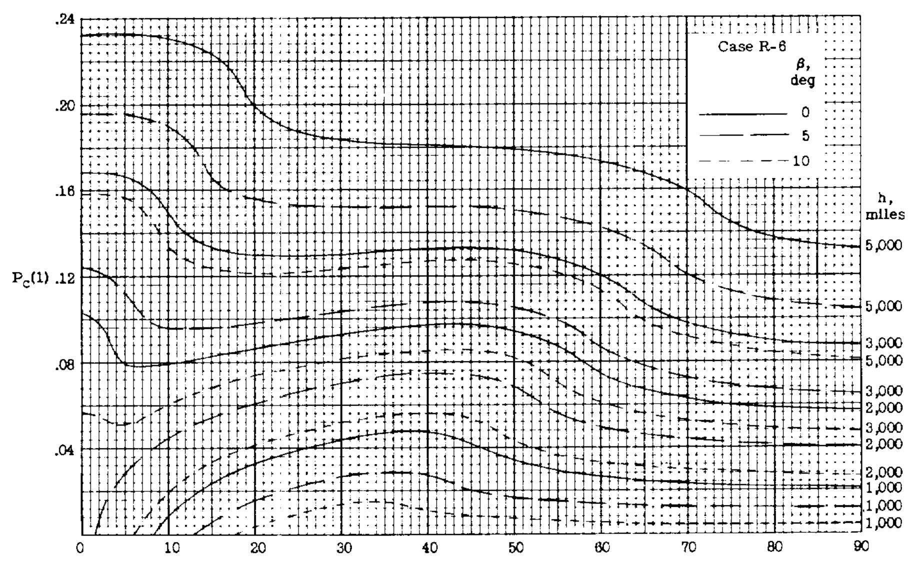

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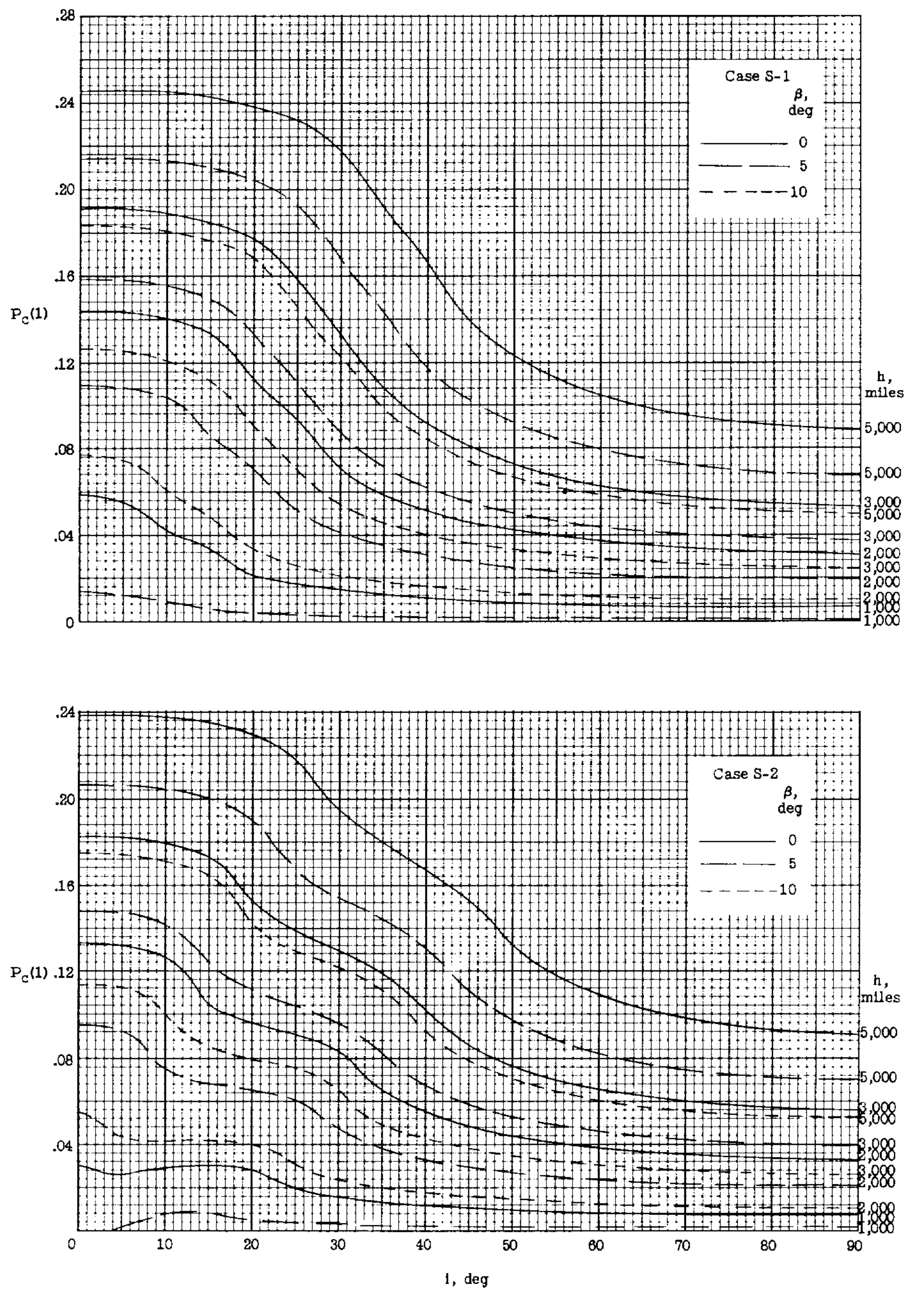

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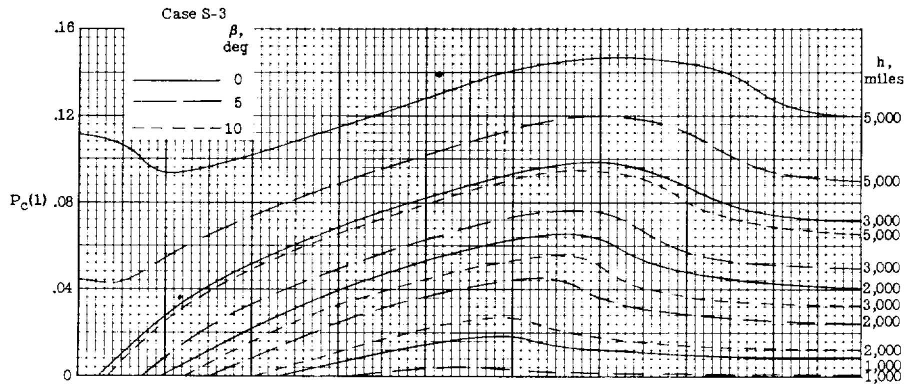

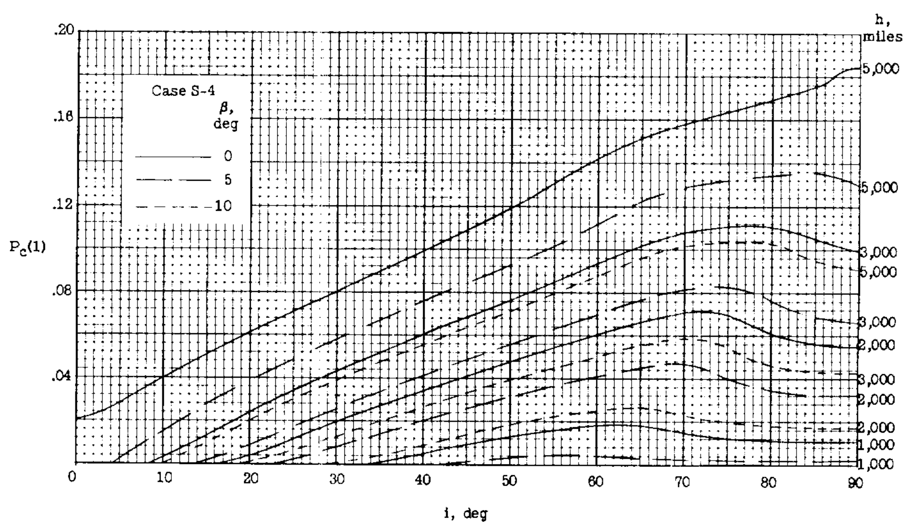

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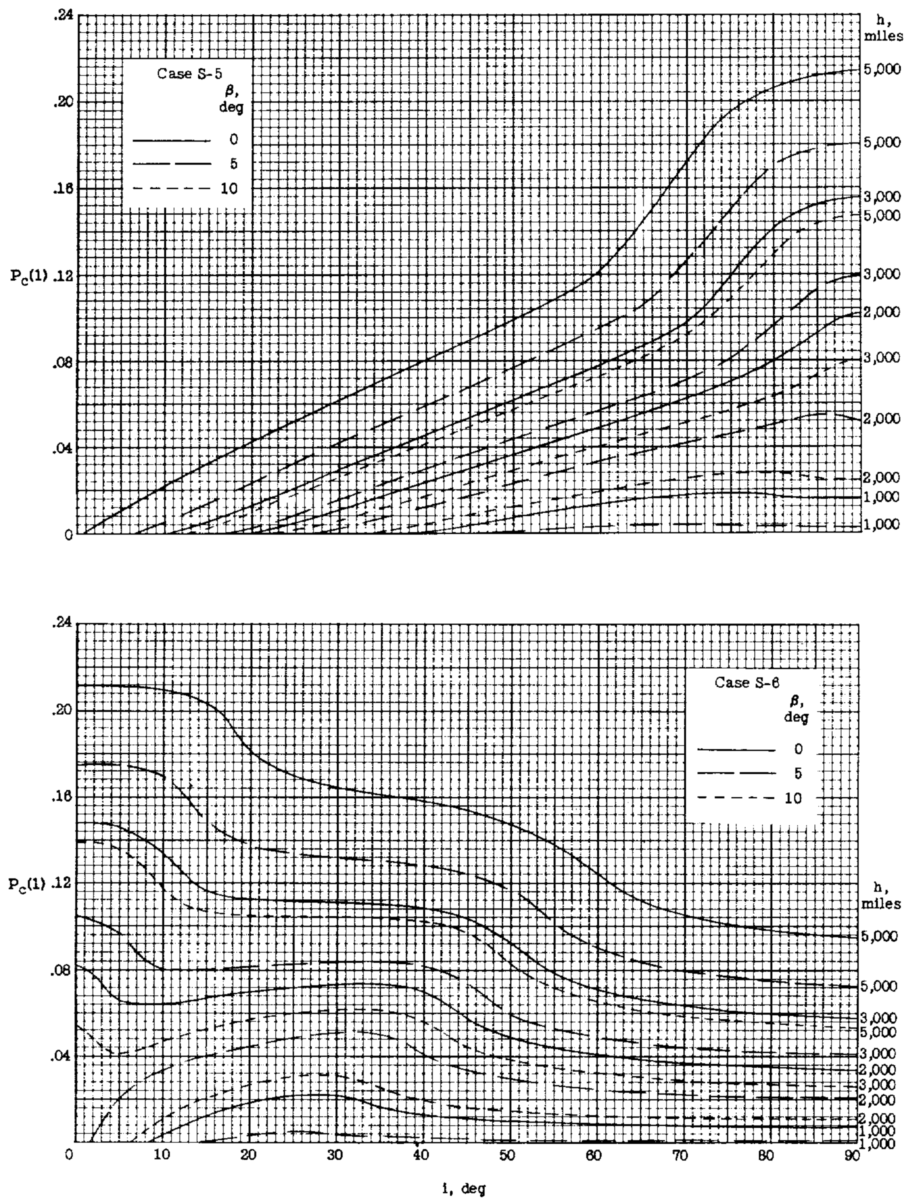

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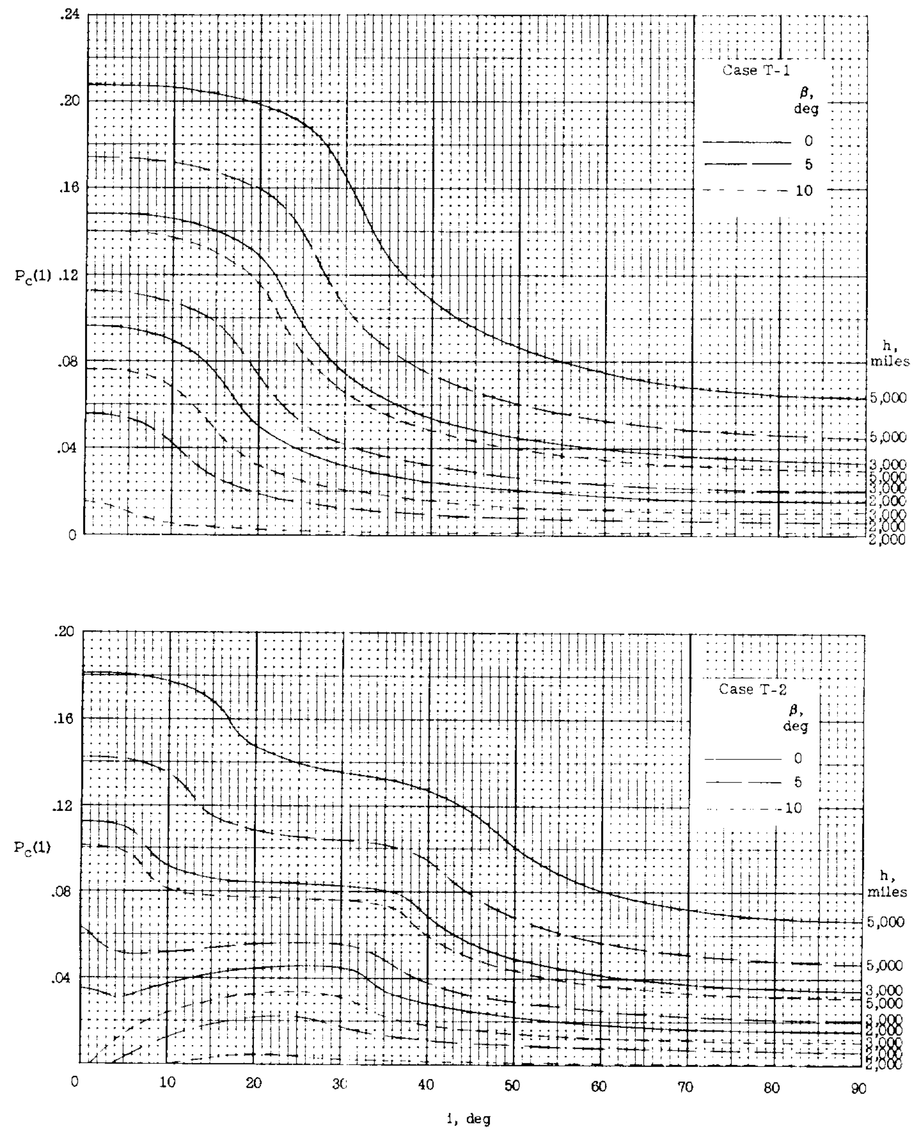

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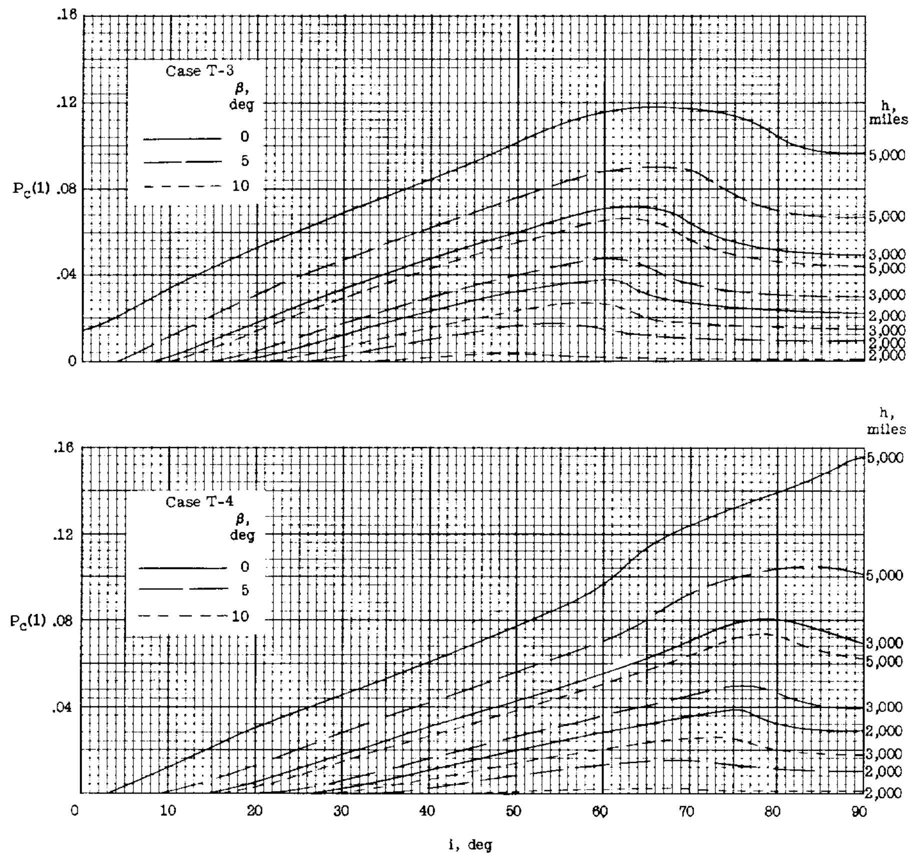

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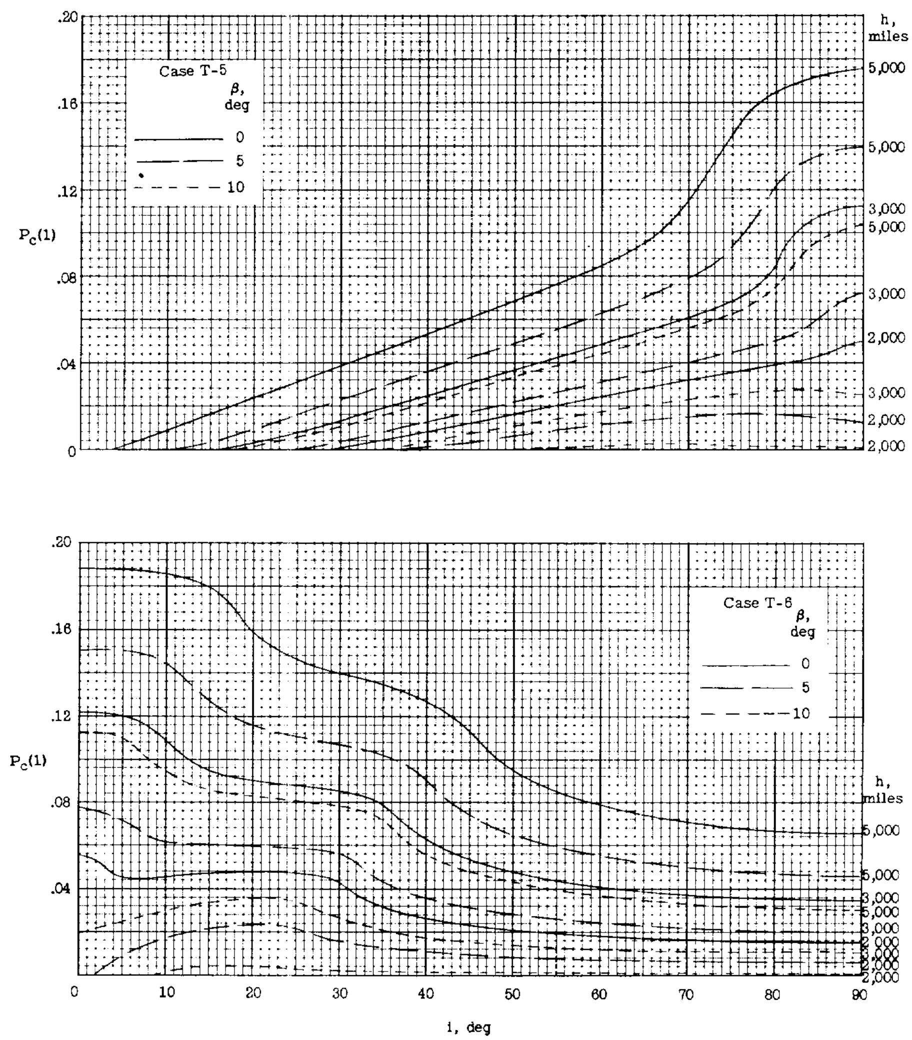

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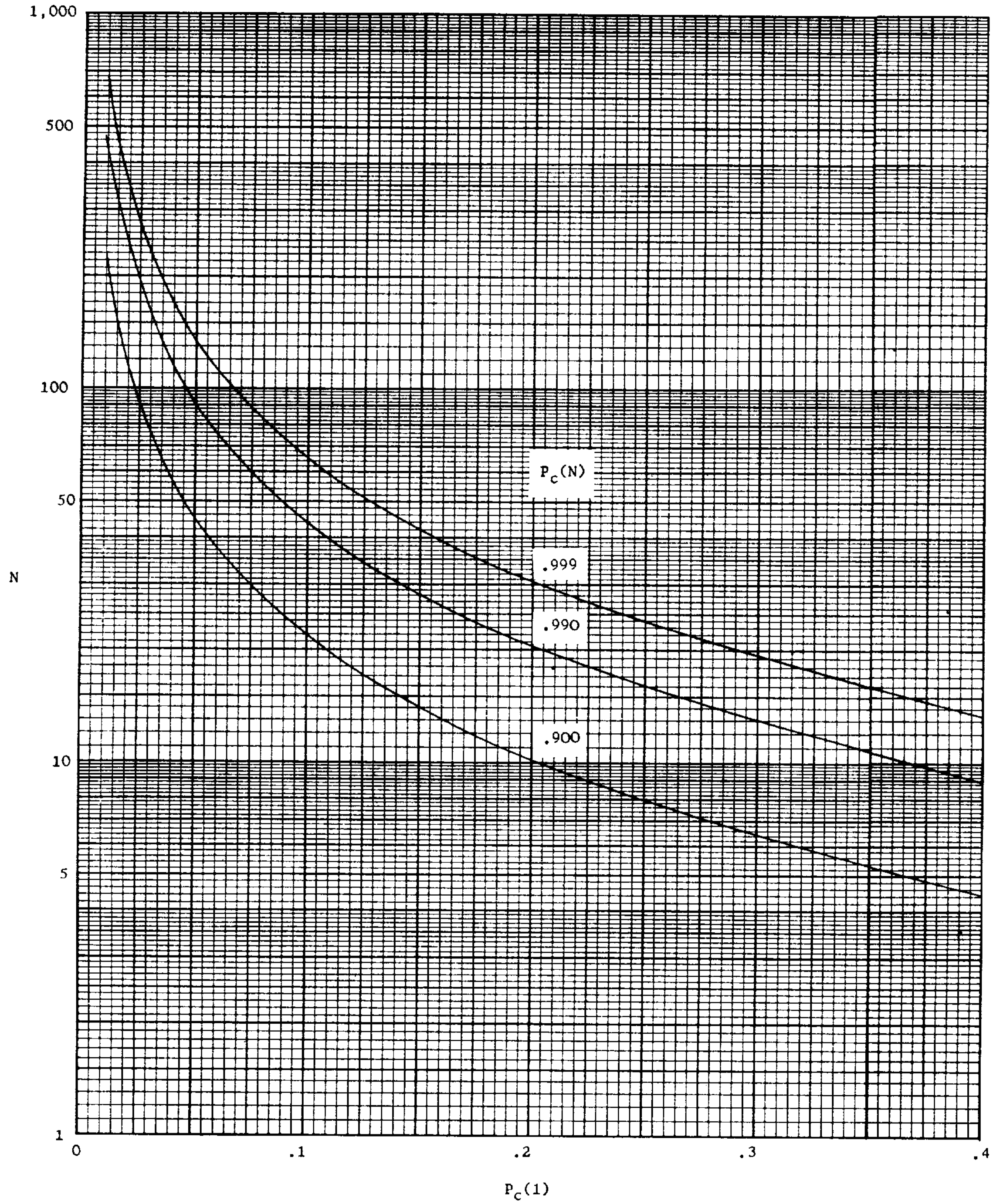
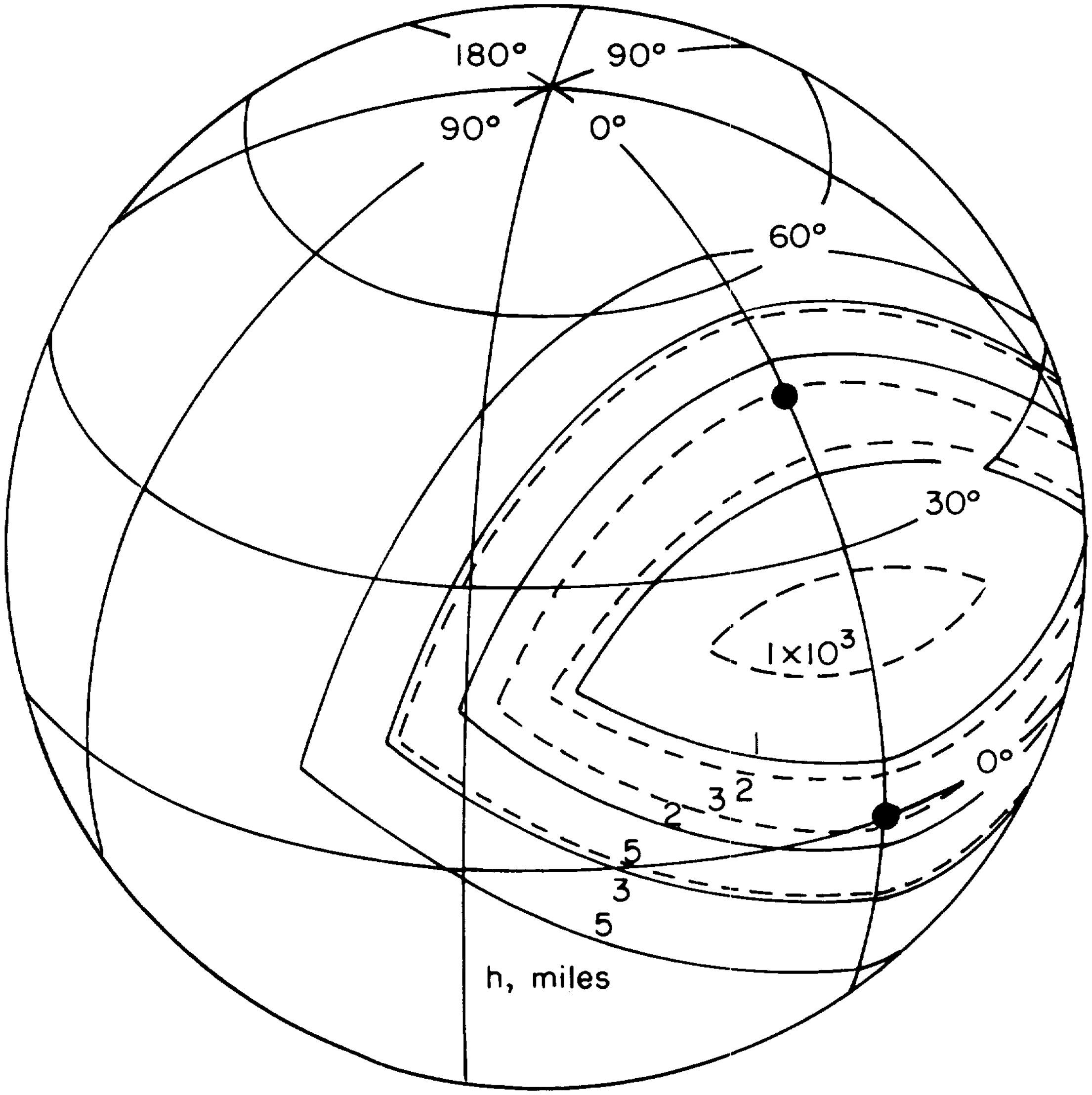


Figure 6.- Relation between number of satellites and $P_c(1)$ for fraction communication time $P_c(N)$.

L-1858



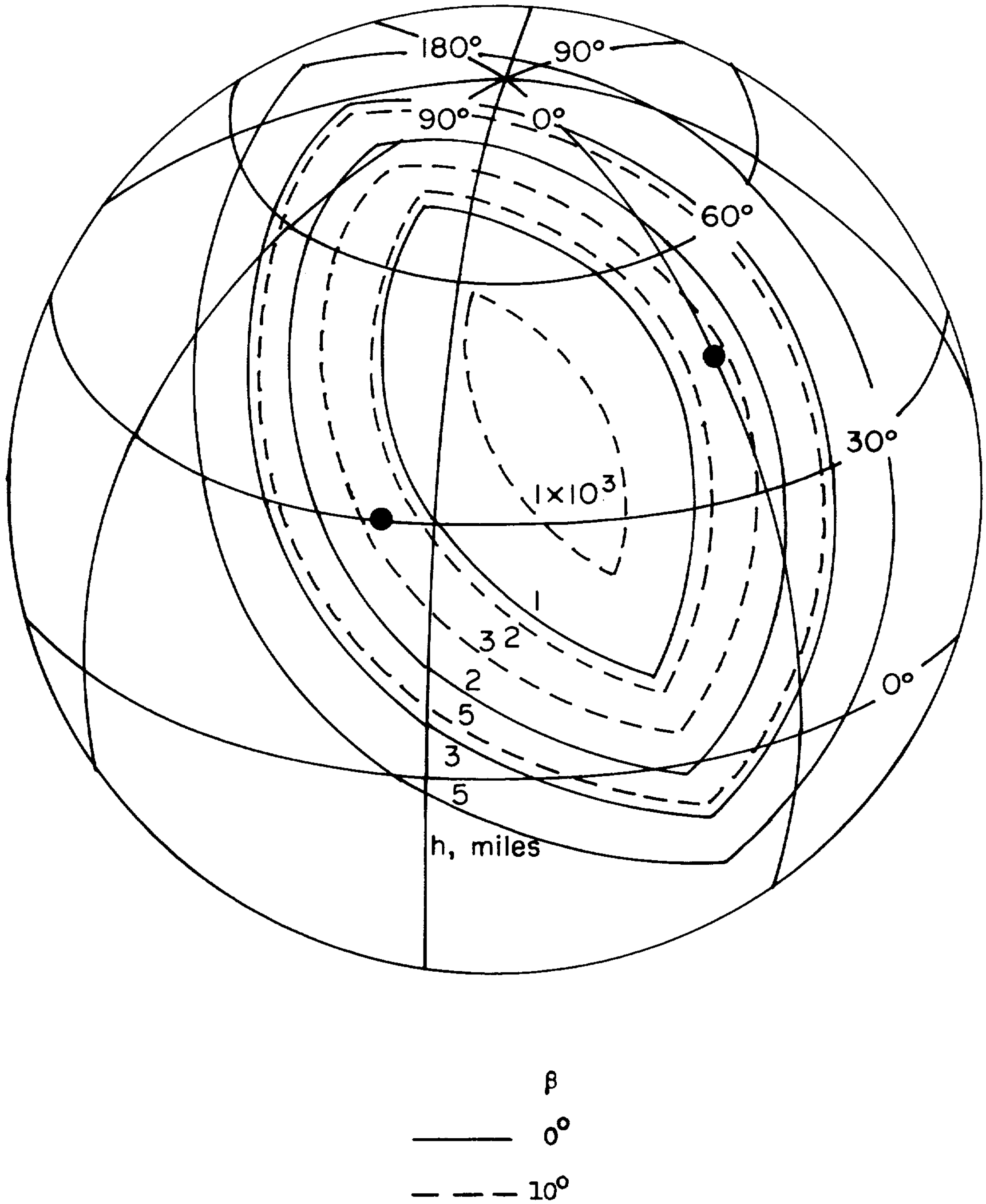
β

0°

10°

(a) Case B-2.

Figure 7.- Sketches of regions of mutual communication.



(b) Case N-1.

Figure 7.- Continued.

L-1858

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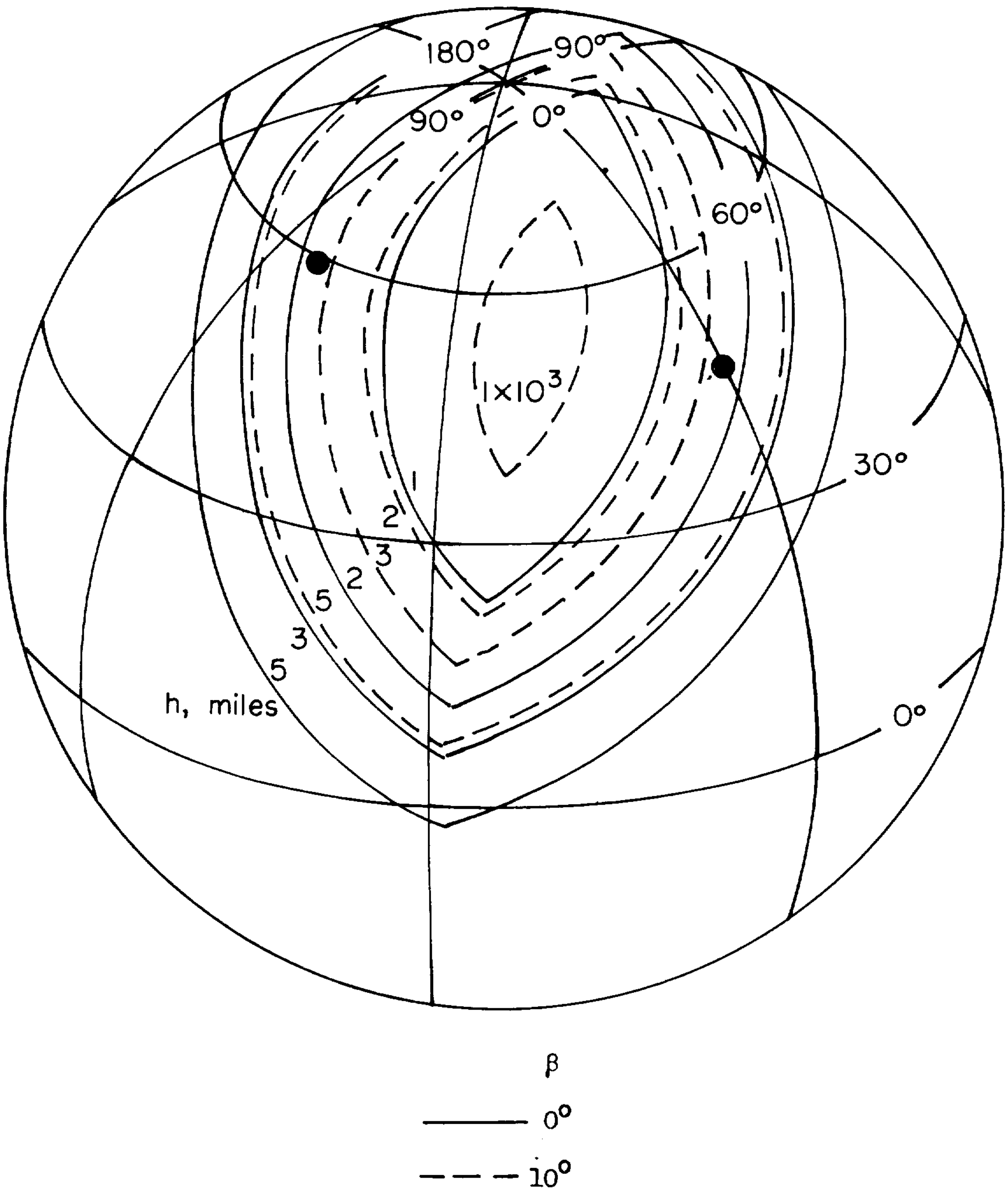


(c) Case R-5.

Figure 7.- Concluded.

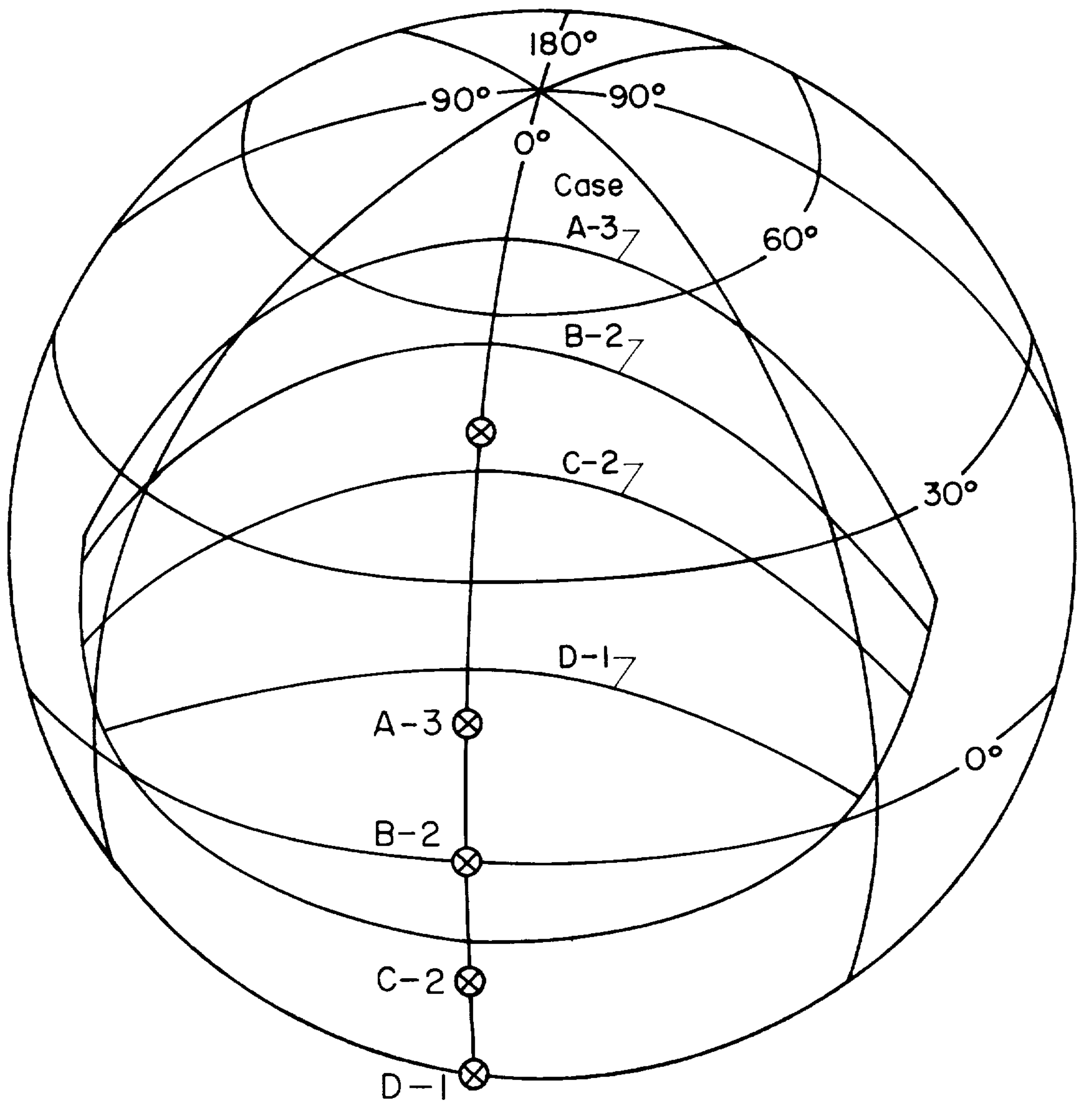


Figure 8.- Regions of mutual communication for cases A-3, B-2, C-2, and D-1. $\theta_d' = 109^{\circ}$.

L-1858

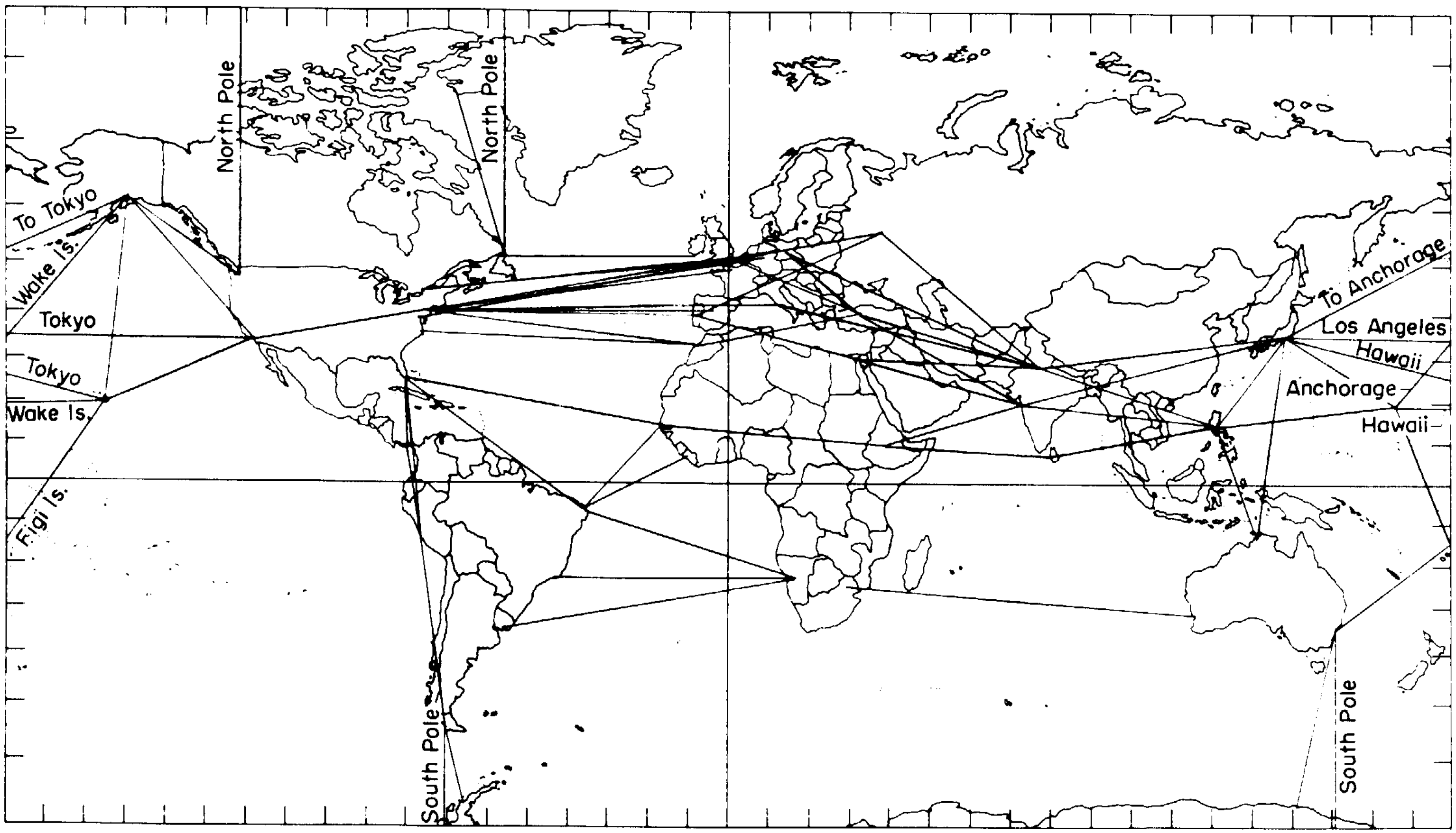


Figure 9.- Map showing sample worldwide communication links.

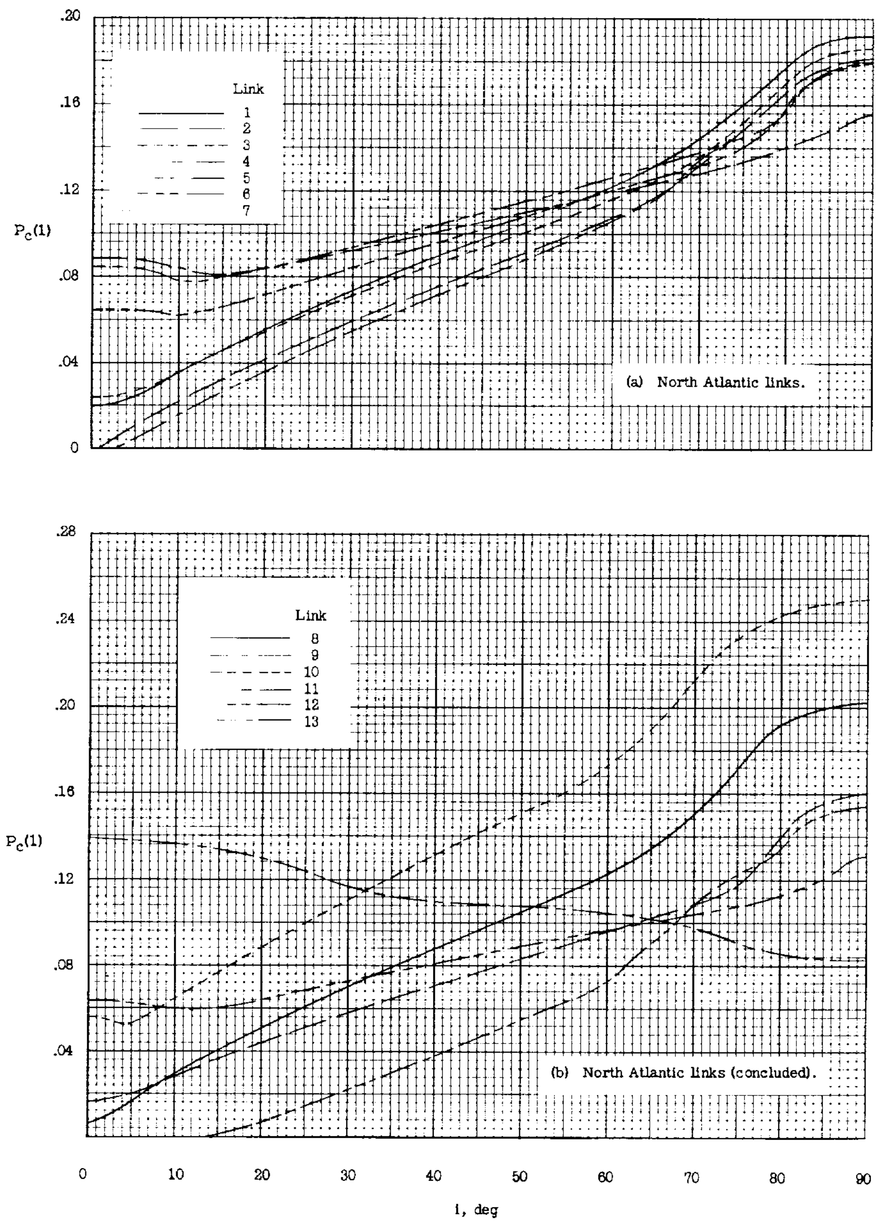


Figure 10.- Variation of probability of communicating having only one satellite in orbit with orbit inclination angle for sample worldwide communications system. $\theta_d' = 118^{\circ}$.

L-1858

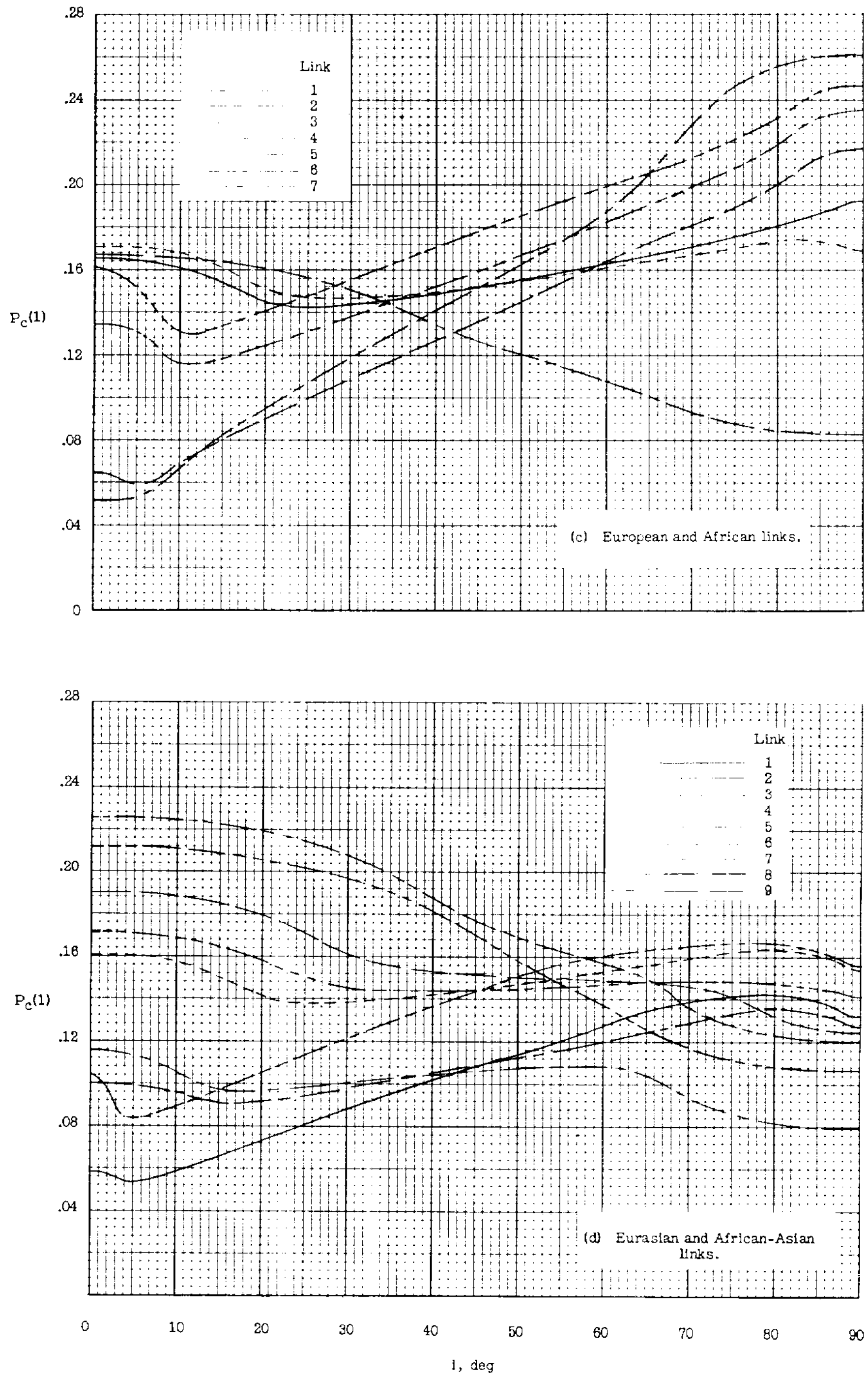
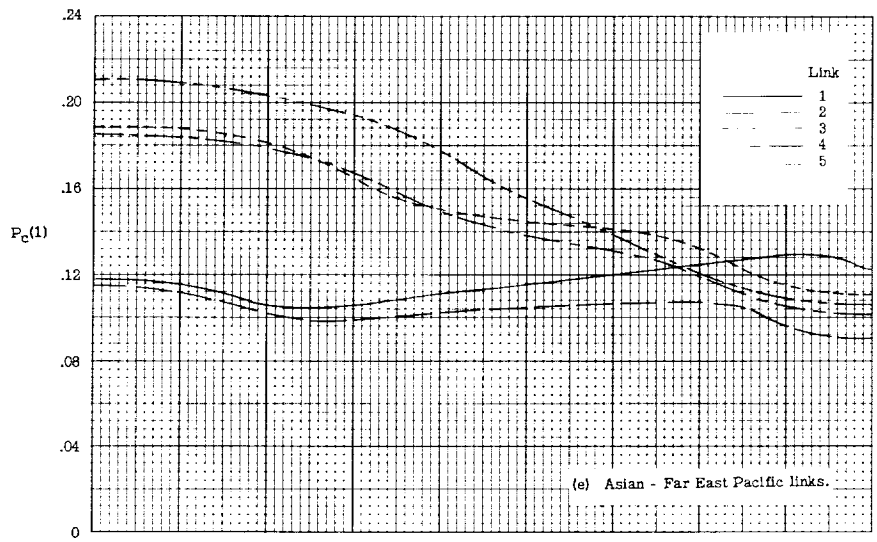
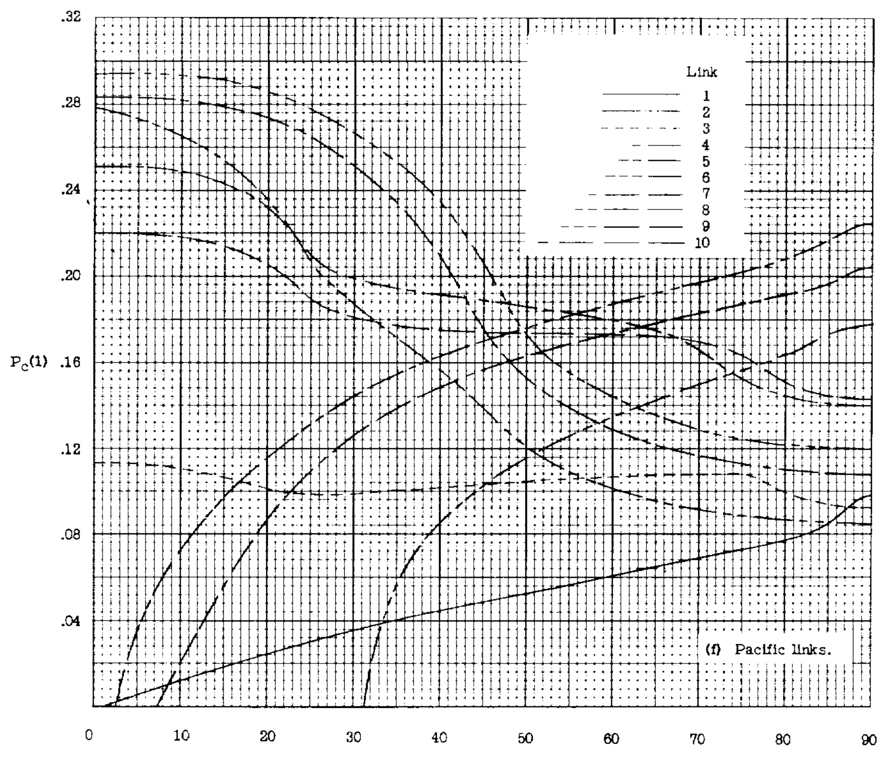


Figure 10.- Continued.



(e) Asian - Far East Pacific links.



(f) Pacific links.

Figure 10.- Continued.

L-1858

L-1858

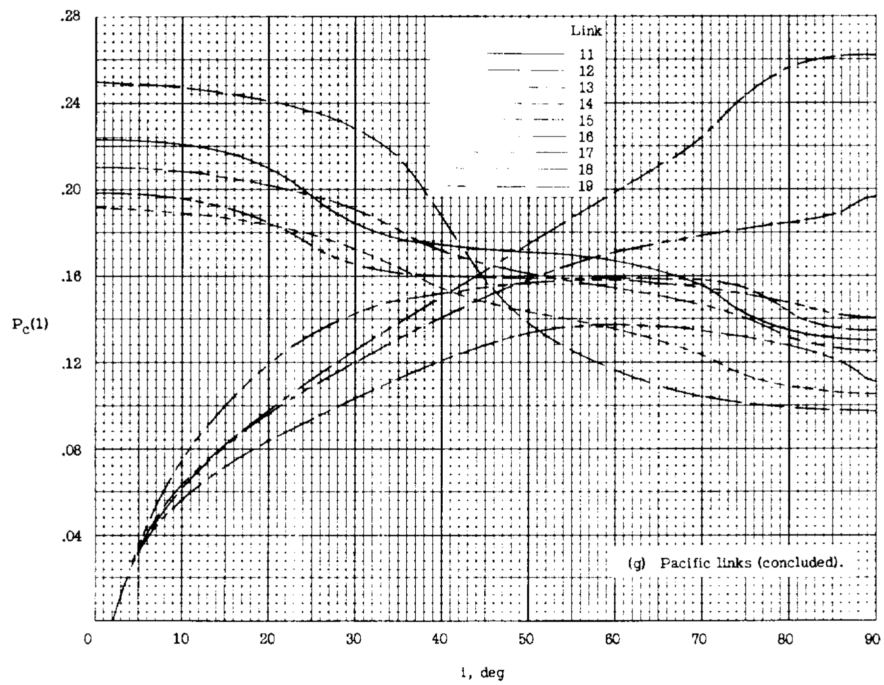


Figure 10.- Continued.

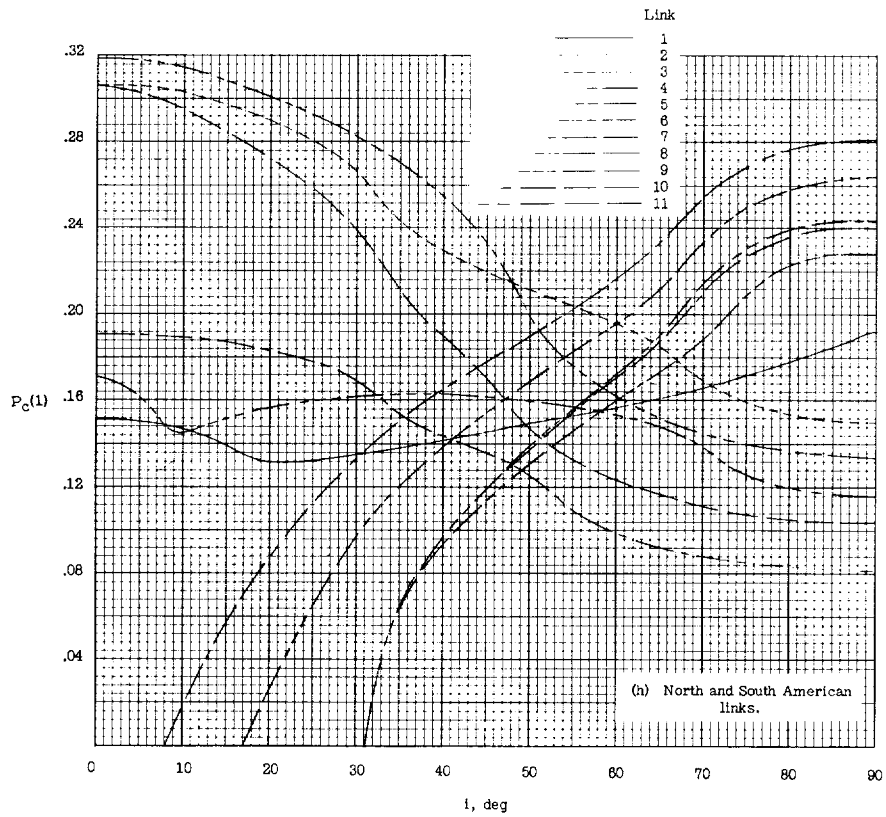


Figure 10.- Continued.

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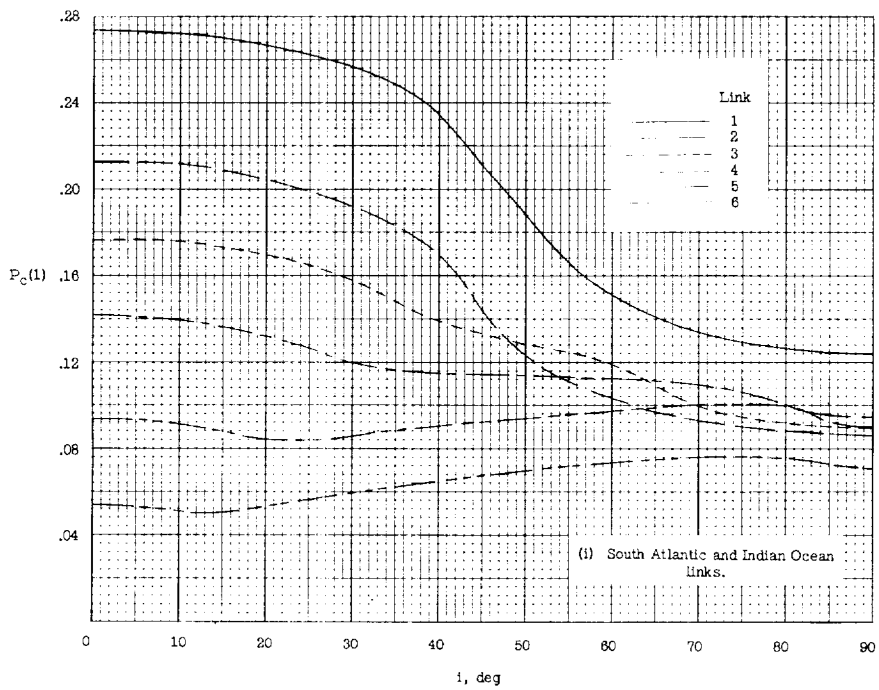
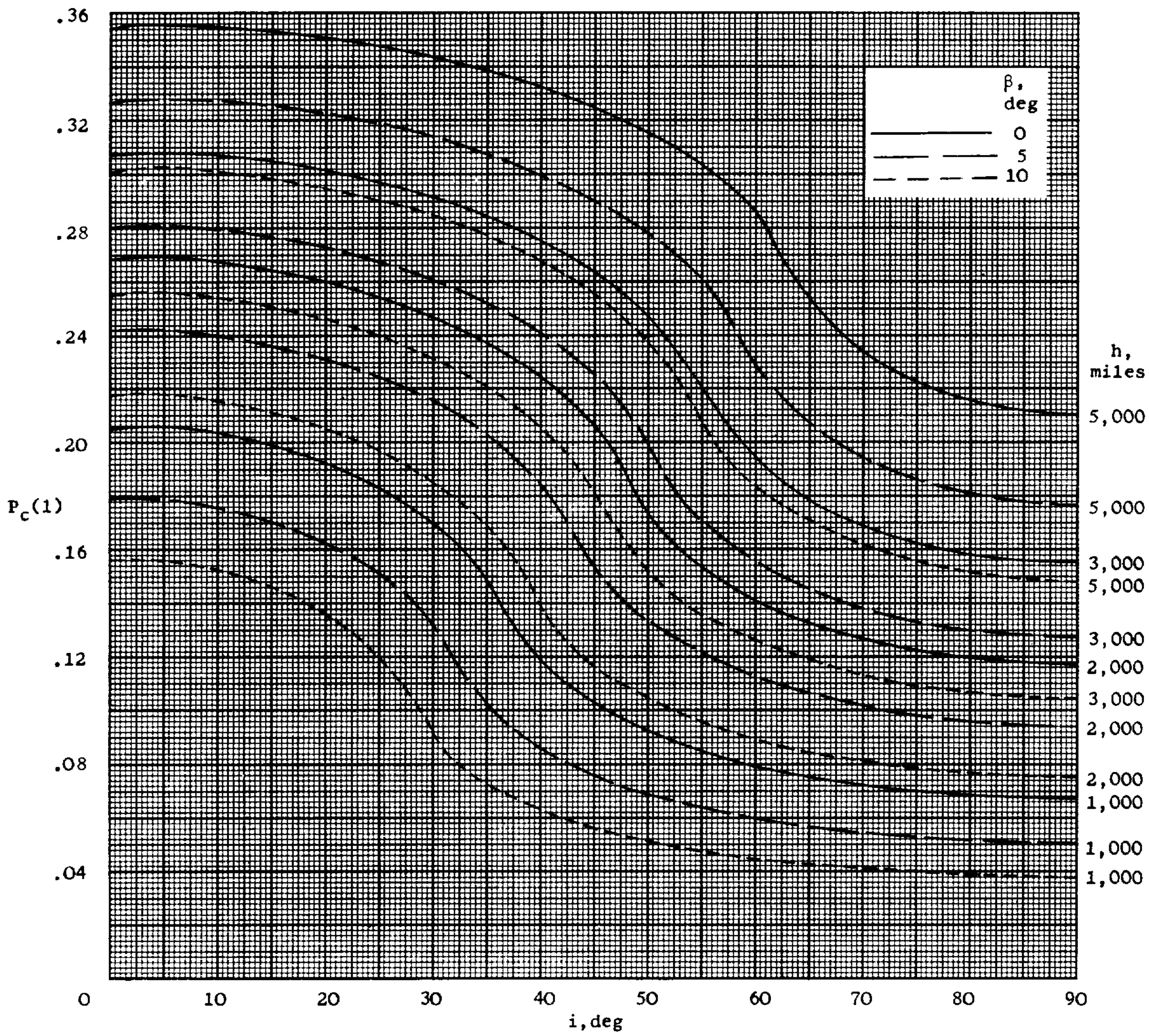


Figure 10.- Concluded.



(a) Latitude, 0°.

Figure 11.- Variation of probability of communicating having only one satellite in orbit with orbit inclination angle for circular region of mutual communication. (Note: These curves apply for combinations of h and β other than the particular ones listed in the figure. See fig. 3.)

L-1858

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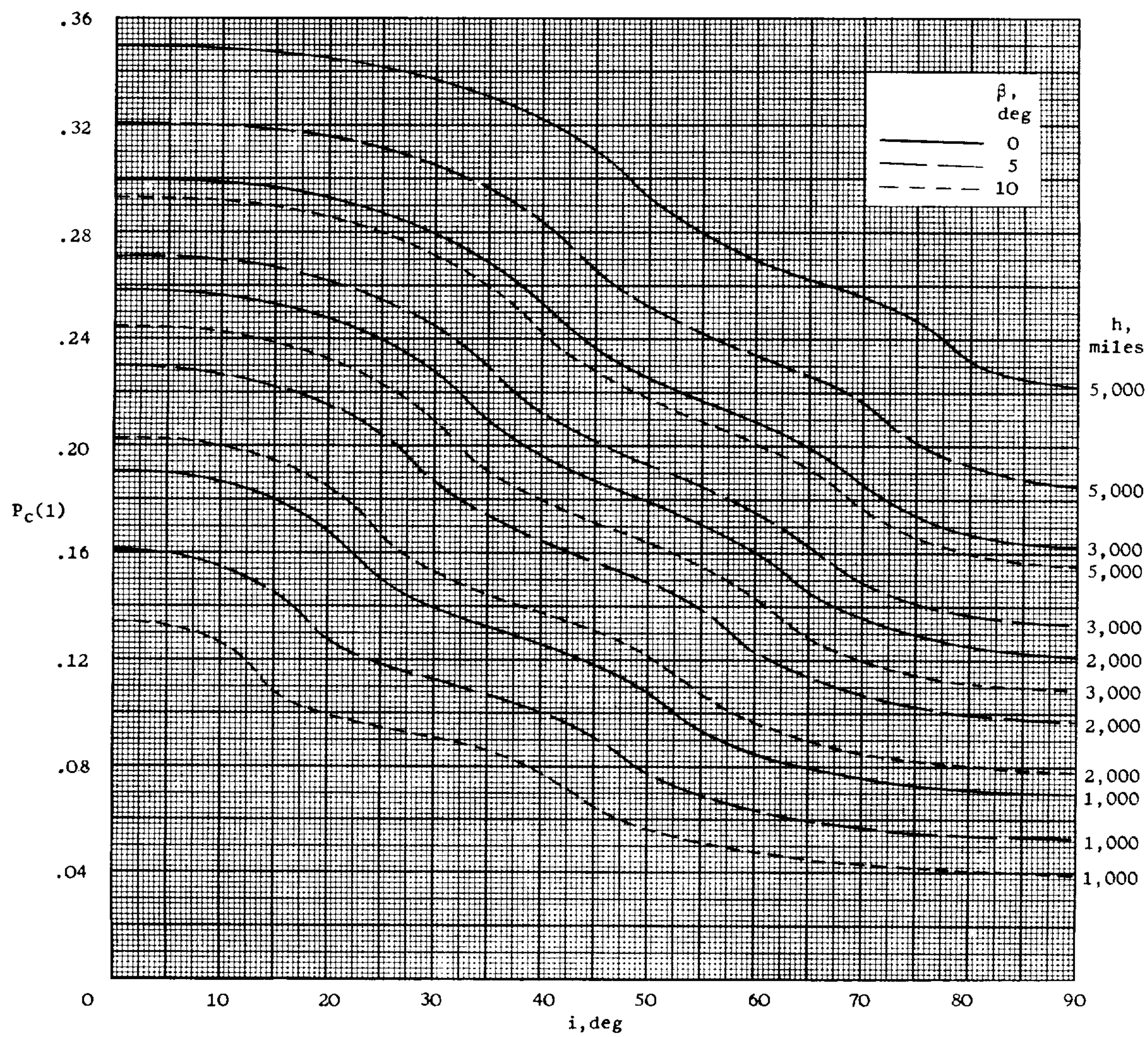
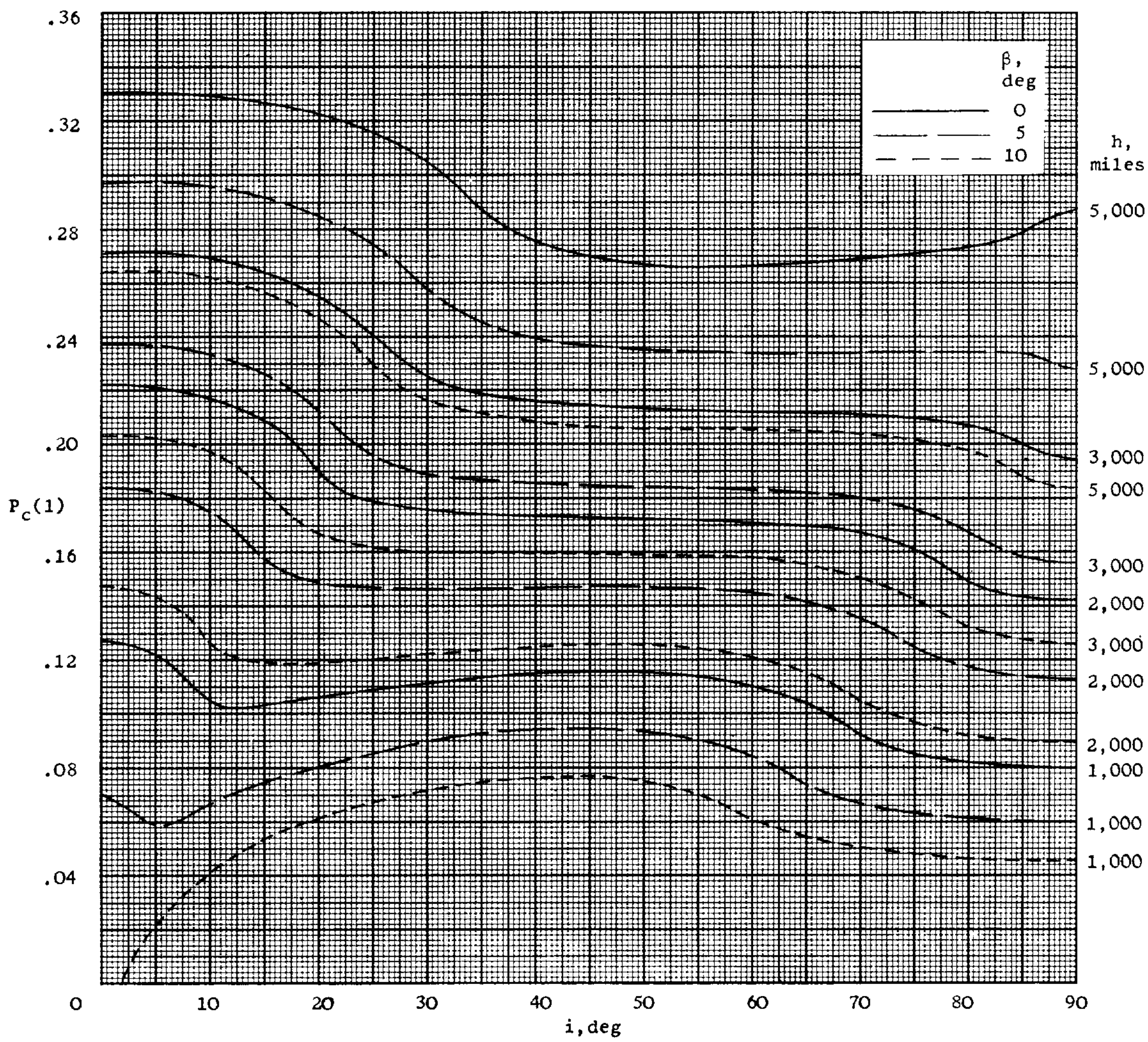


(b) Latitude, 15°.

Figure 11.- Continued.



(c) Latitude, 30°.

Figure 11.- Continued.

L-1858

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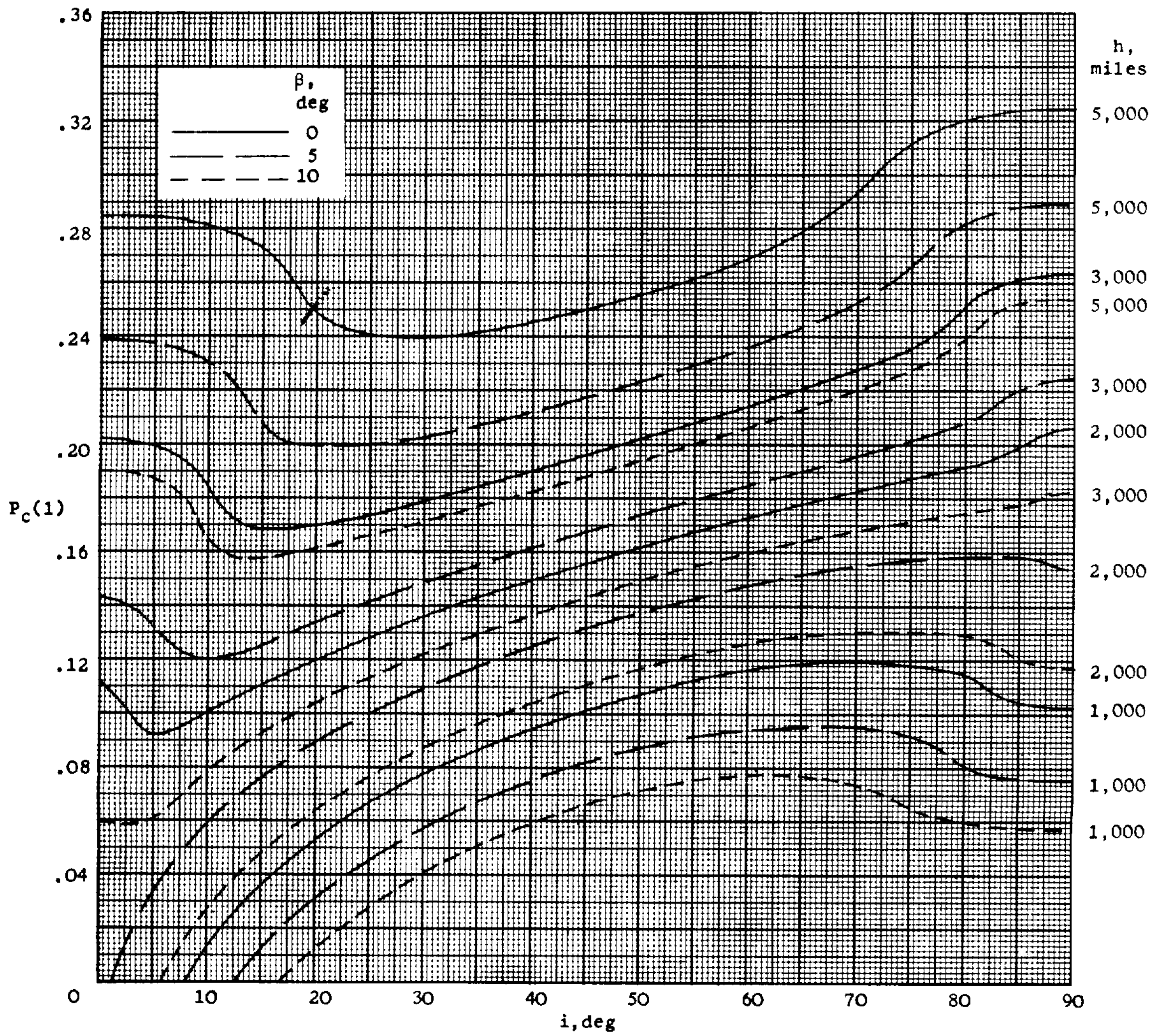
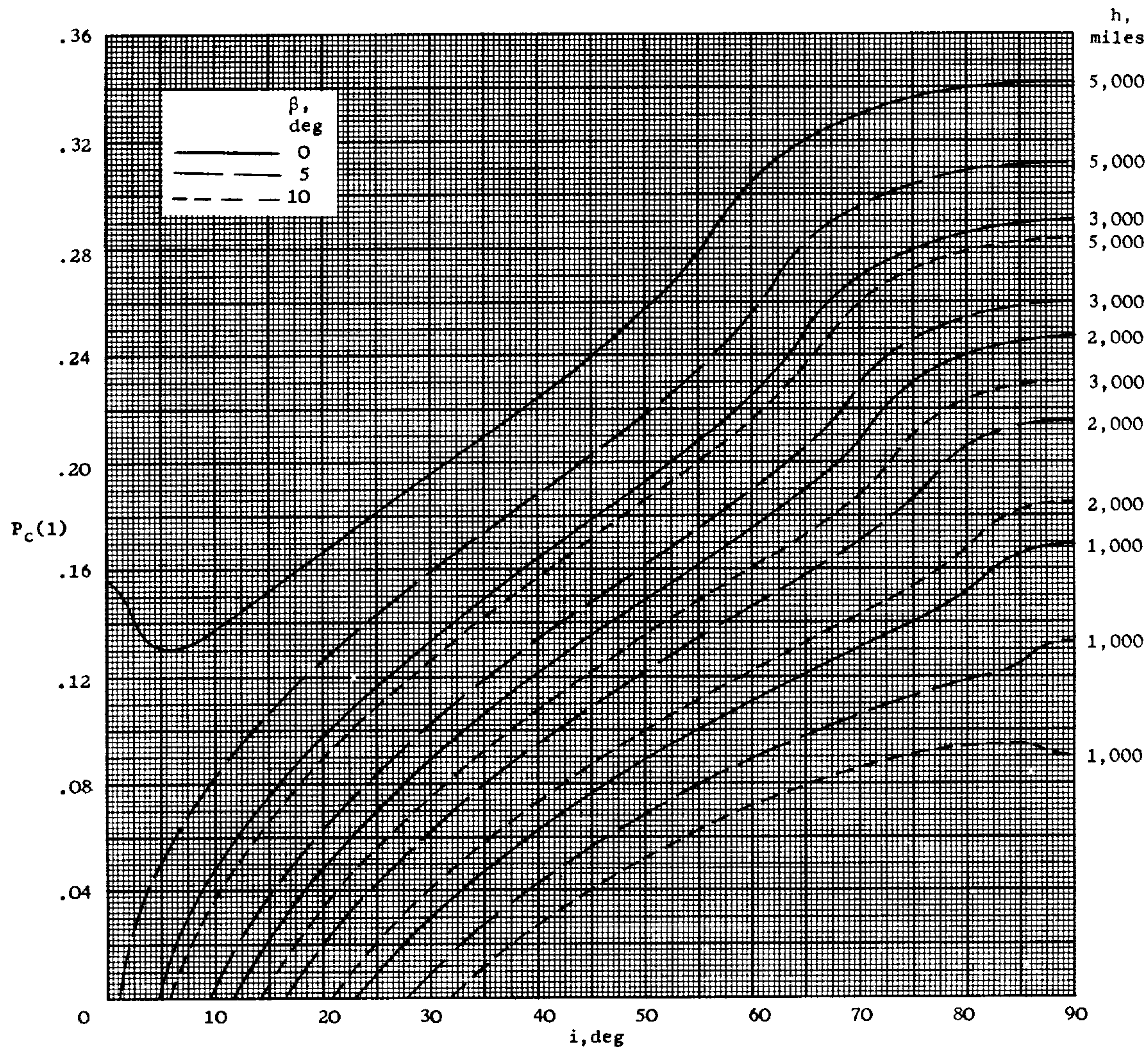


(d) Latitude, 45°.

Figure 11.- Continued.



(e) Latitude, 60°.

Figure 11.- Continued.

L-1858

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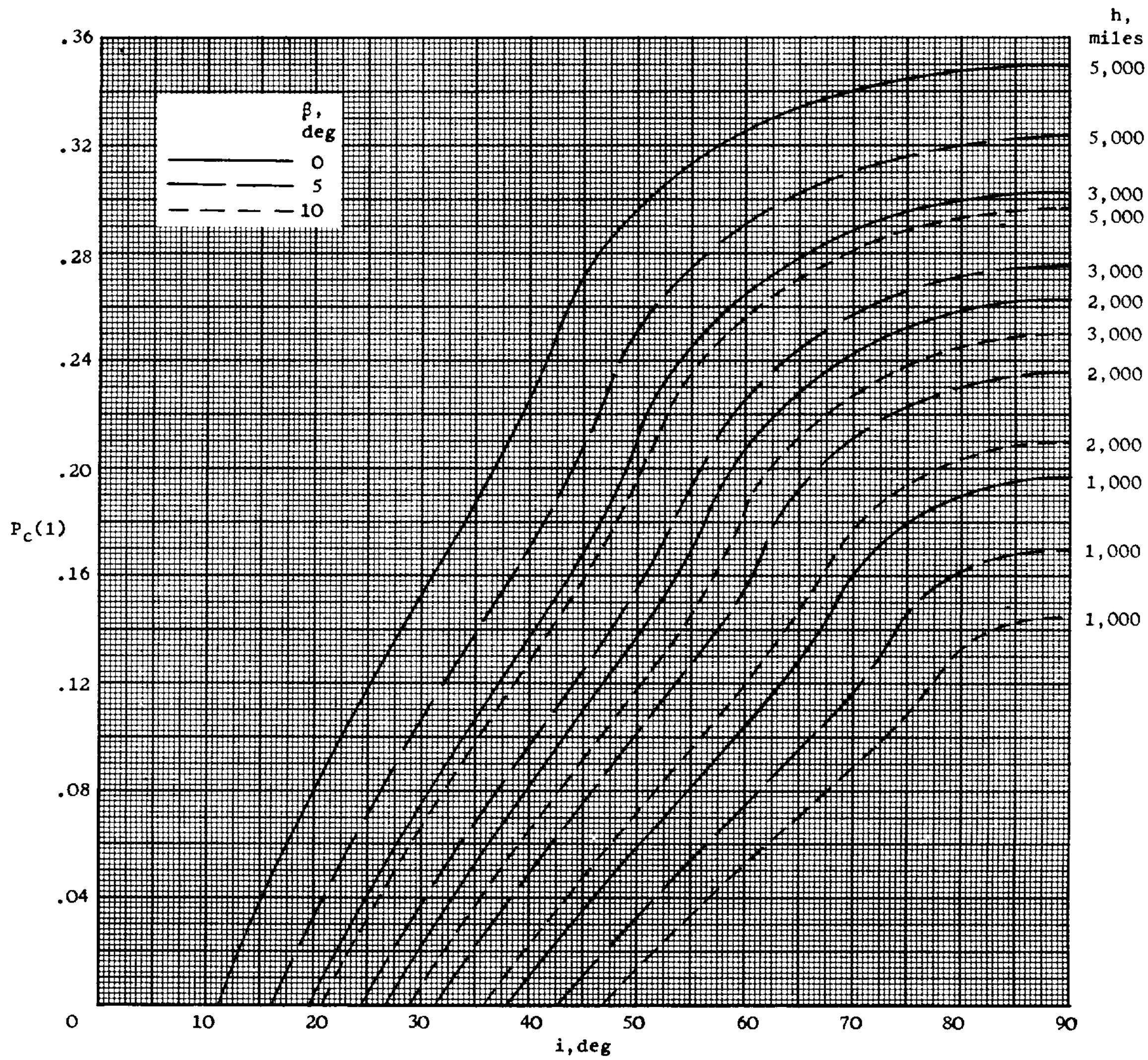
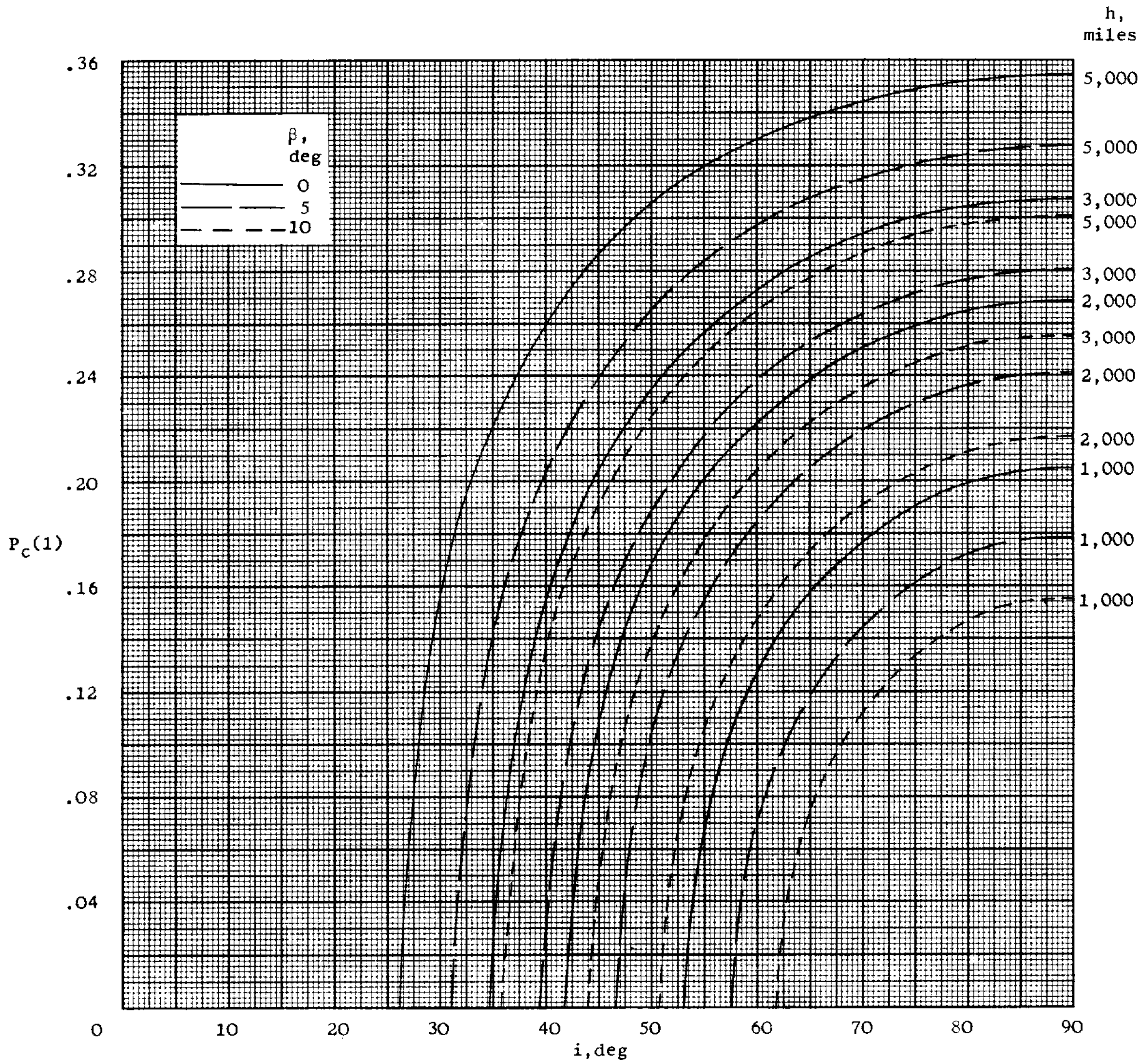


(f) Latitude, 75°.

Figure 11.- Continued.



(g) Latitude, 90°.

Figure 11.- Concluded.

L-1858

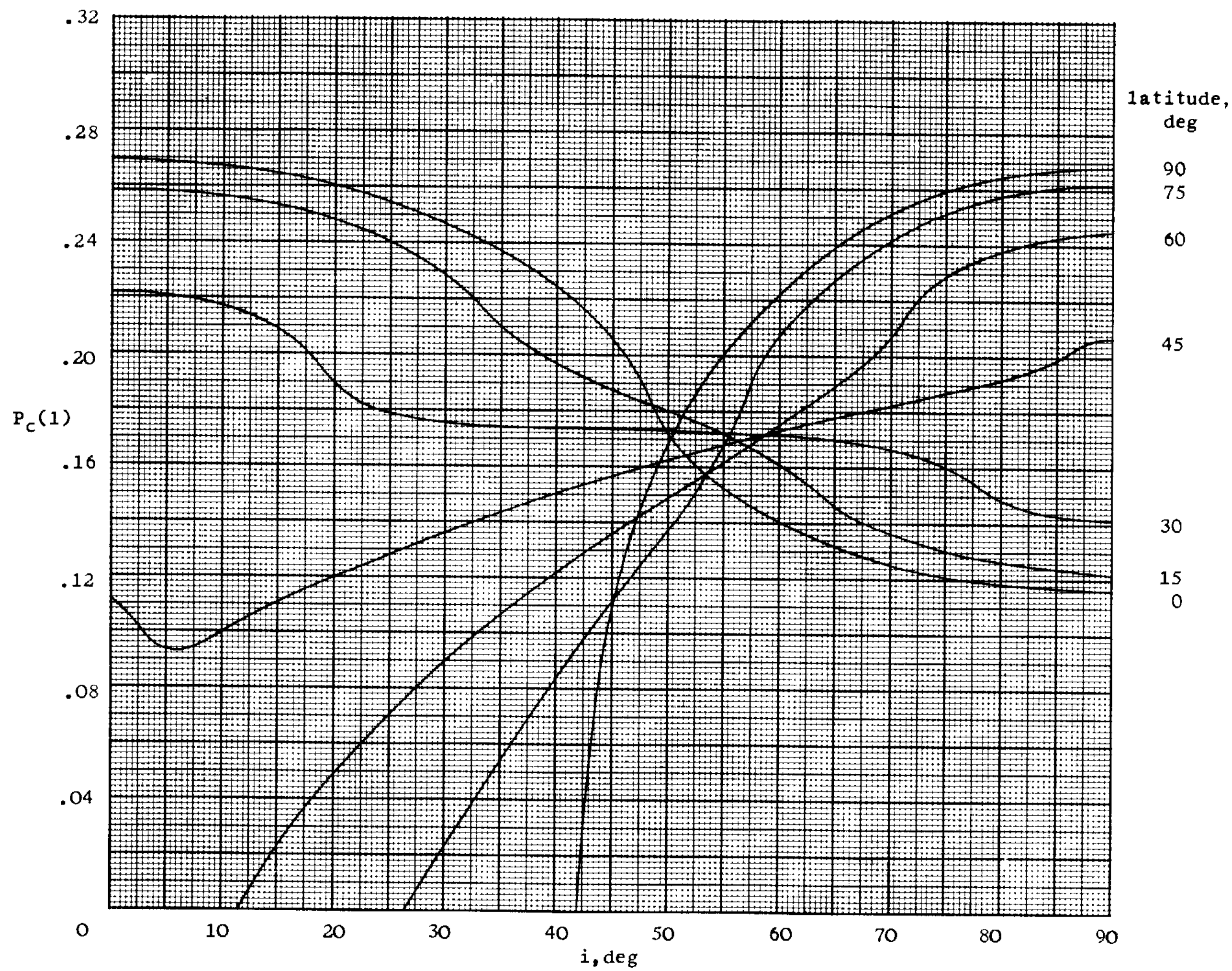


Figure 12.- Sample plot for determining i_{opt} for communicating from a station at any latitude, based on circular region of mutual communication. $\theta_d' = 96^{\circ}$; h = 2,000 miles, $\beta = 0^{\circ}$.

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